A Primer for G. Spencer Brown's Laws of Form

by
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Mark this!

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Acknowledgements

This book came about as the result of a remarkable lecture that a physicist from the University of Florida, Dr. Joseph Rosenshein, gave at the Fourth Lexington Conference on Trialectics. His splendid introduction to the Laws of Form gave me the first glimmering that I could understand them.

He has continued to serve as a source of my understanding throughout the process of developing this book.

I also want to acknowledge the helpful conversations with Norman Hirst, a philosopher and logician at Holodynamics in Redding,Connecticut. His understanding of self-reference added much to my own.

Also thanks to Hal Caswell, a population biologist at the Woods Hole Oceanographic Institution for valuable discussions on the philosophy of science and for checking over the biological sections of this manuscript.

And thanks to Susan Osgood, my typist, who had to depart from the normal to type things in baloons, and keep track of the unmarked state!

Chapter 1

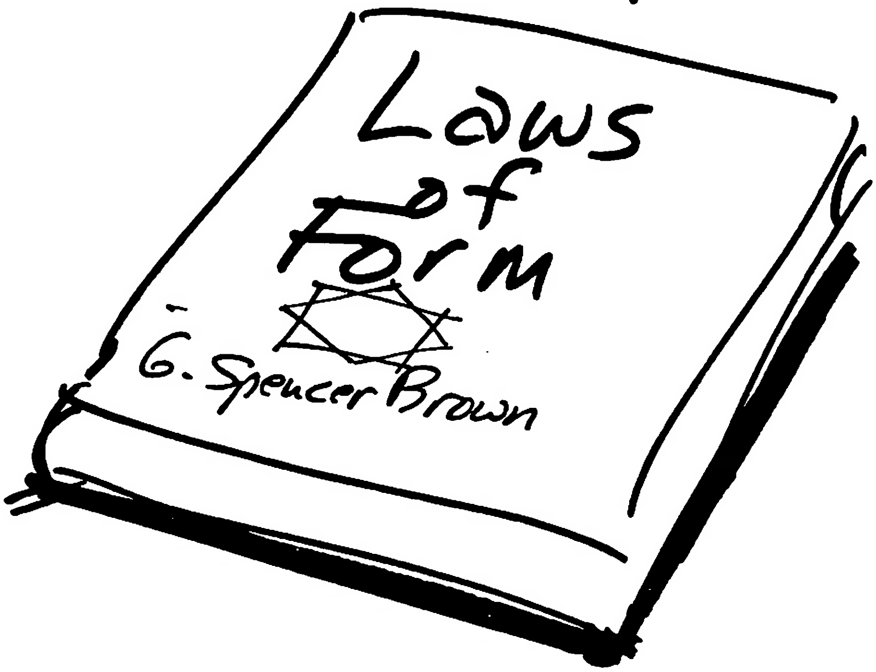
Distinctions

What are They?

Why is The study of them important?

In

a remarkable book was published...

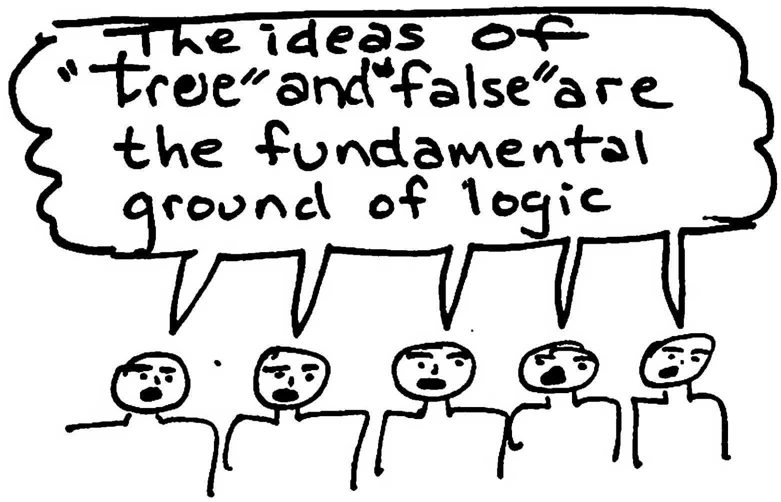


by a relatively unknown Oxford don named G. Spencer Brown

Bertrand Russell said

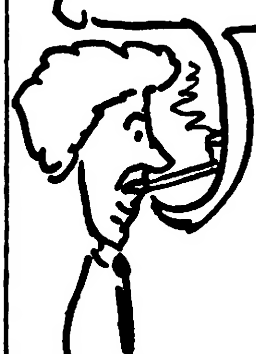
In this book Mr Spencer Brown has succeeded in doing what, in mathematics, is very rare indeed. He has revealed a new calculus, of great power and simplicity. I congratulate him

Until the time that Spencer Brown published Laws of Form, it was felt that



In 1919,
Bertrand Russell
had said

The problem is: "What are the constituents of a logical proposition?" I do not know the answer, but I propose to explain how the problem arises. . . . We may accept as a first approximation, the view that *forms* are what enter into logical propositions as their constituents. And we may explain (though not formally define) what we mean by the form of a proposition as follows: The form of a proposition is that, in it, that remains unchanged when every constituent of the proposition is replaced by another. (Russell, 1919:128)



But nobody had thoroughly investigated the Laws of those forms until Spencer Brown.

Heinz Von Foerster, one of the pioneering figures in the new field of cybernetics, wrote this review of Laws of Form.

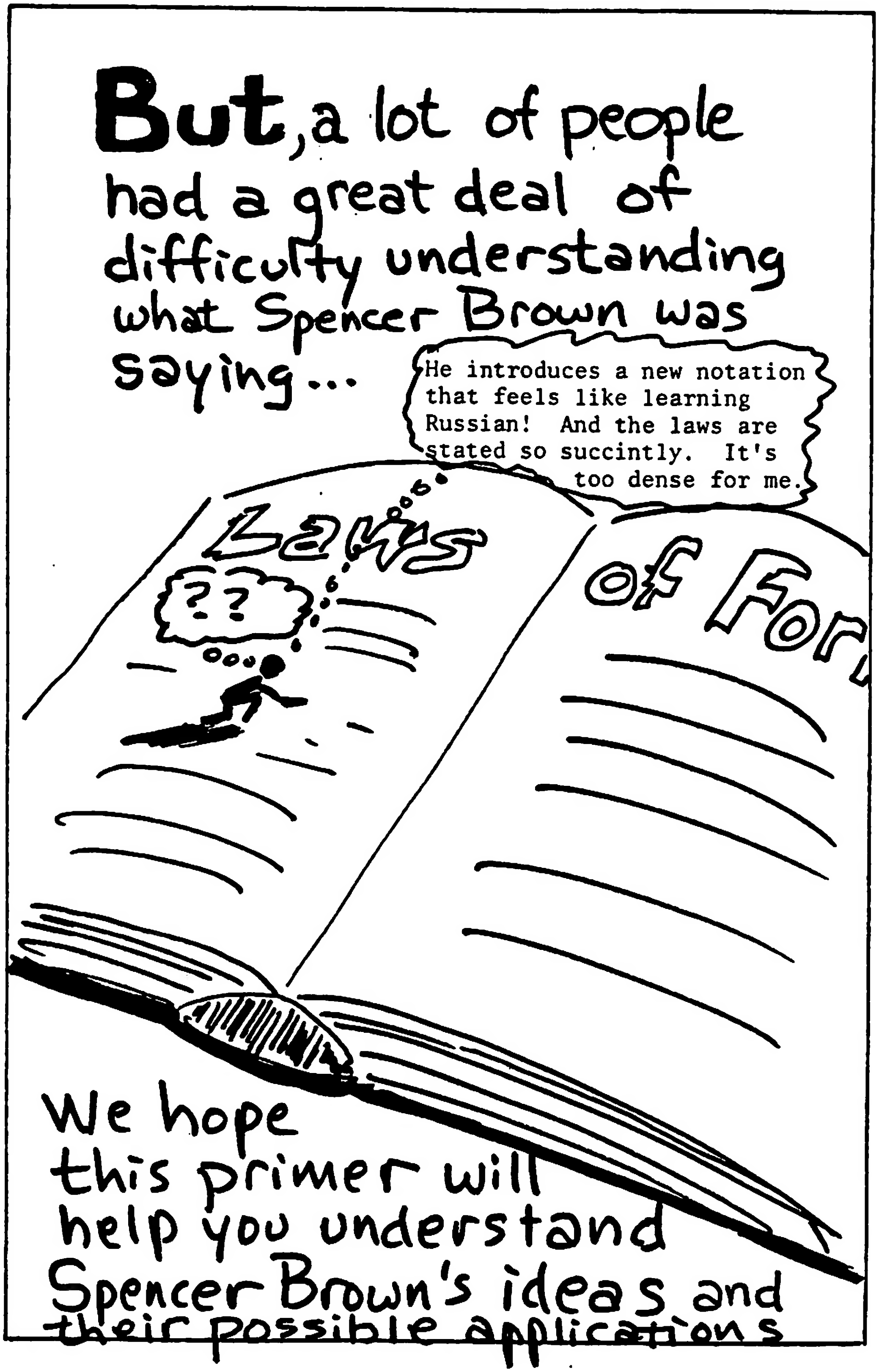
The laws of form have finally been written! With a "Spencer Brown" transistorized power razor (a Twentieth Century model of Occam's razor), G. Spencer Brown cuts smoothly through two millennia of growth of the most prolific and persistent of semantic weeds, presenting us with his superbly written *Laws of Form.* This Herculean task which now, in retrospect, is of profound simplicity rests on his discovery of the form of laws. Laws are not descriptions, they are commands, injunctions: "Do!" Thus, the first constructive proposition in this book (page 3) is the injunction: "Draw a distinction!" an exhortation to perform the primordial creative act.

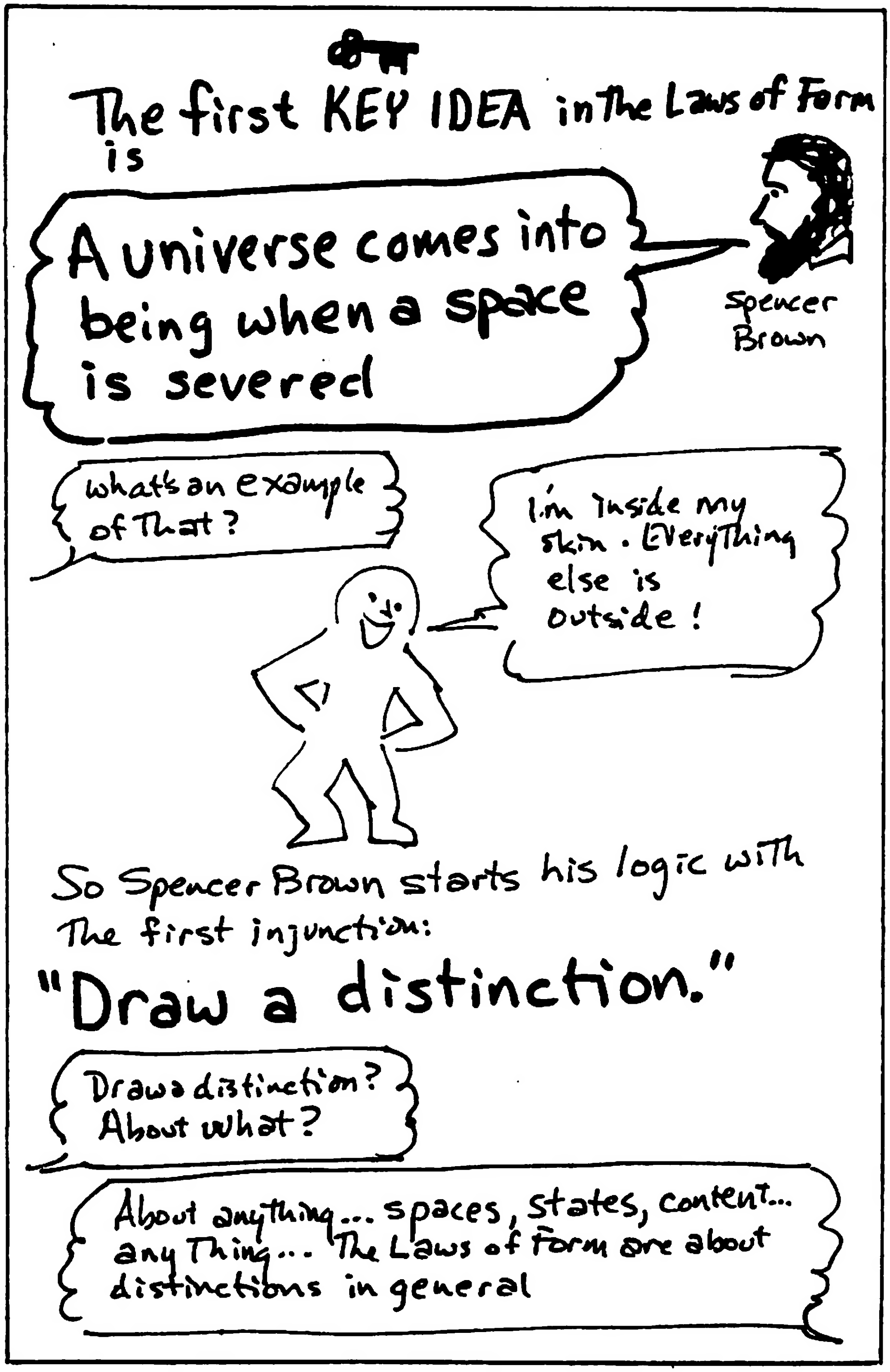
After this, practically everything else follows smoothly: a rigorous foundation of arithmetic, of algebra, of logic, of a calculus of indications, intentions and desires; a rigorous development of laws of form, may they be of logical relations, of descriptions of the universe by physicists and cosmologists, or of functions of the nervous system which generates descriptions of the universe of which it is itself a part.

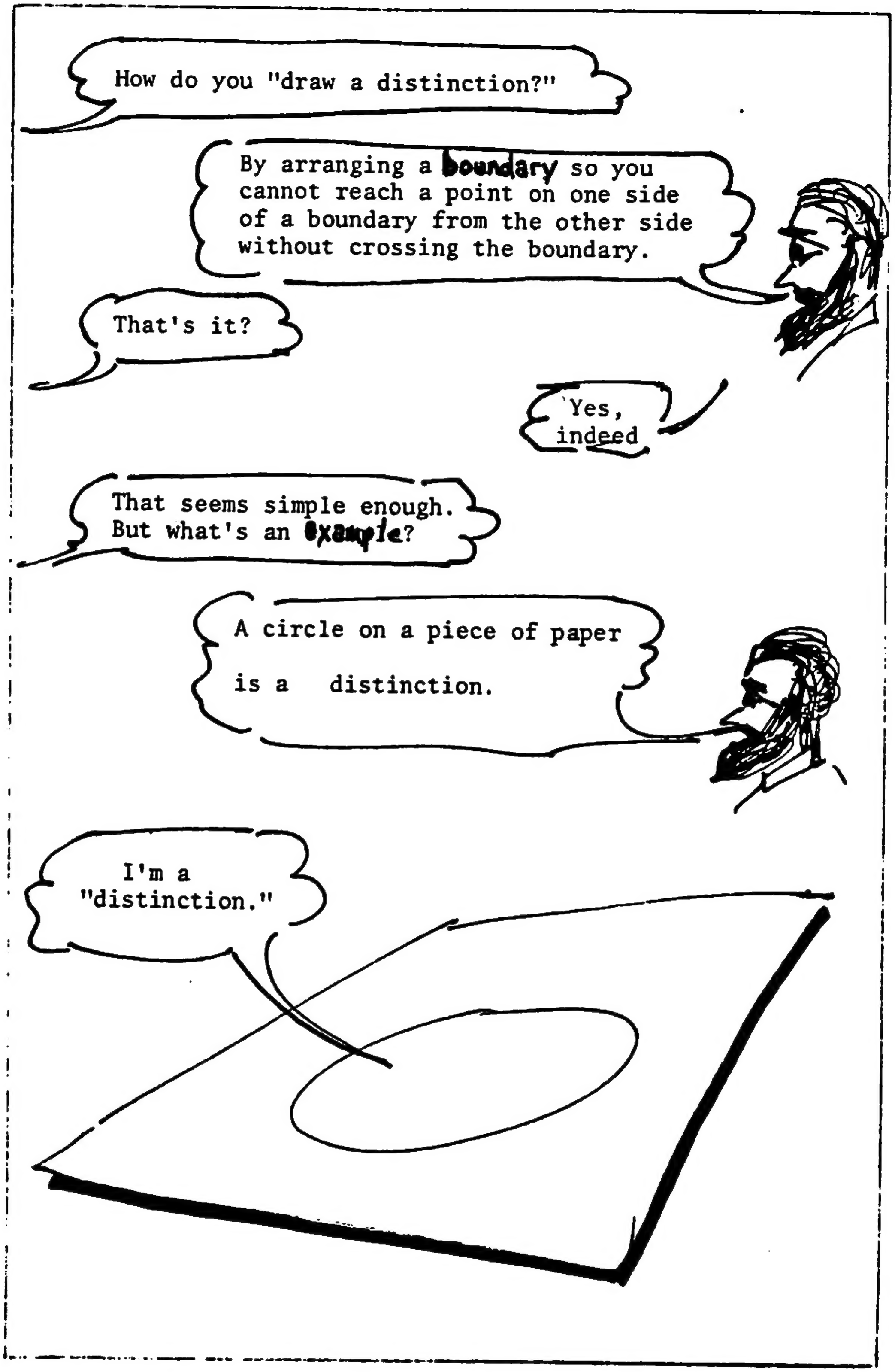
The ancient and primary mystery which still puzzled Ludwig Wittgenstein (*Tractatus Logico-Philosophicus,* A. J. Ayer (ed), Humanities Press, New York, 1961, 166 pp.), namely that the world we know is constructed in such a way as to be able to see itself, G. Spencer Brown resolves by a most surprising turn of perception. He shows, once and for all, that the appearance of this mystery is unavoidable. But what is unavoidable is, in one sense, no mystery. The fate of all descriptions is ". . . what is revealed will be concealed, but what is concealed will again be revealed."

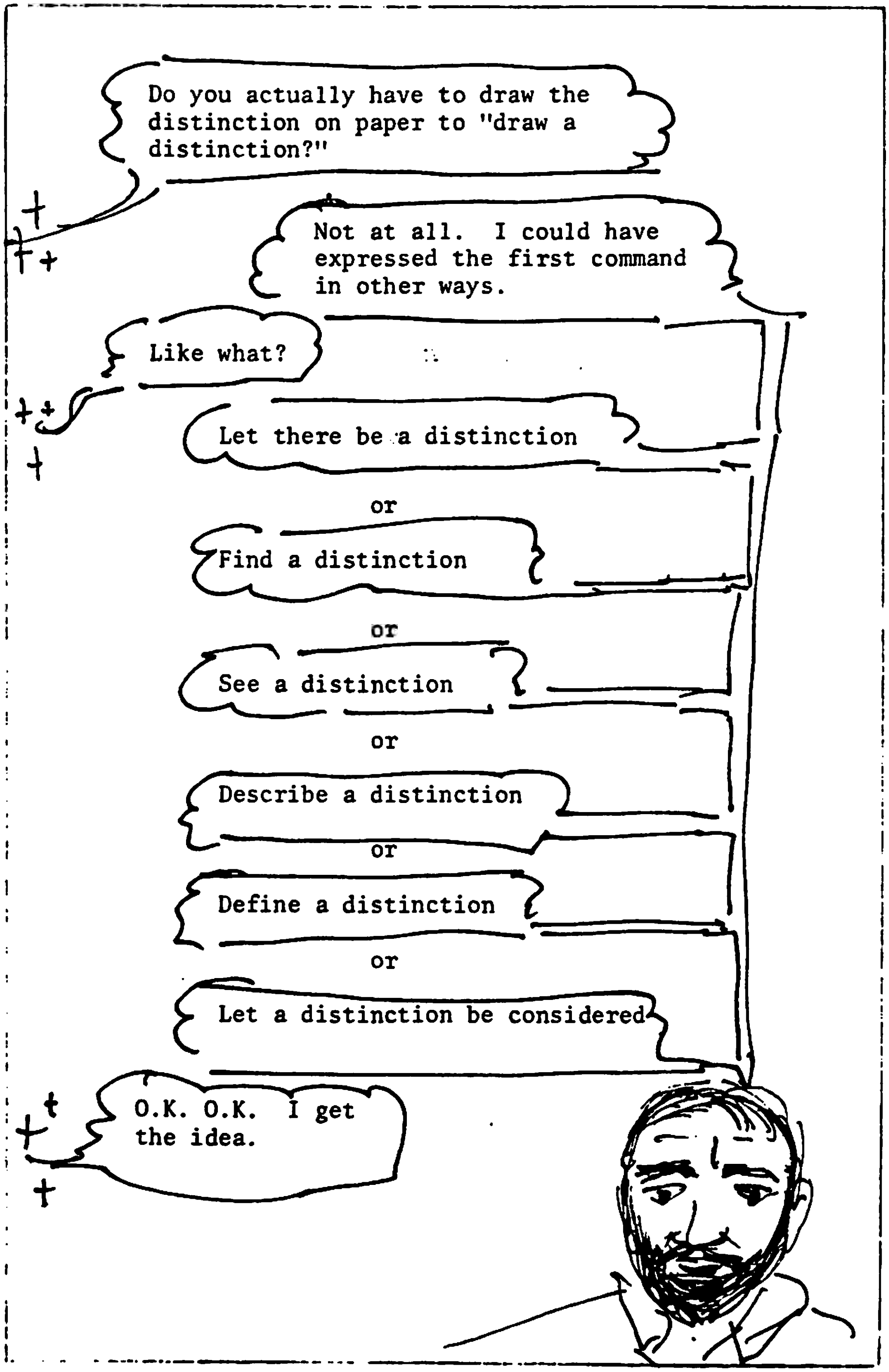
At this point, even the most faithful reader may turn suspicious: how can the conception of such a simple injunction as "Draw a distinction!" produce this wealth of insights? It is indeed amazing—but, in fact, it does.

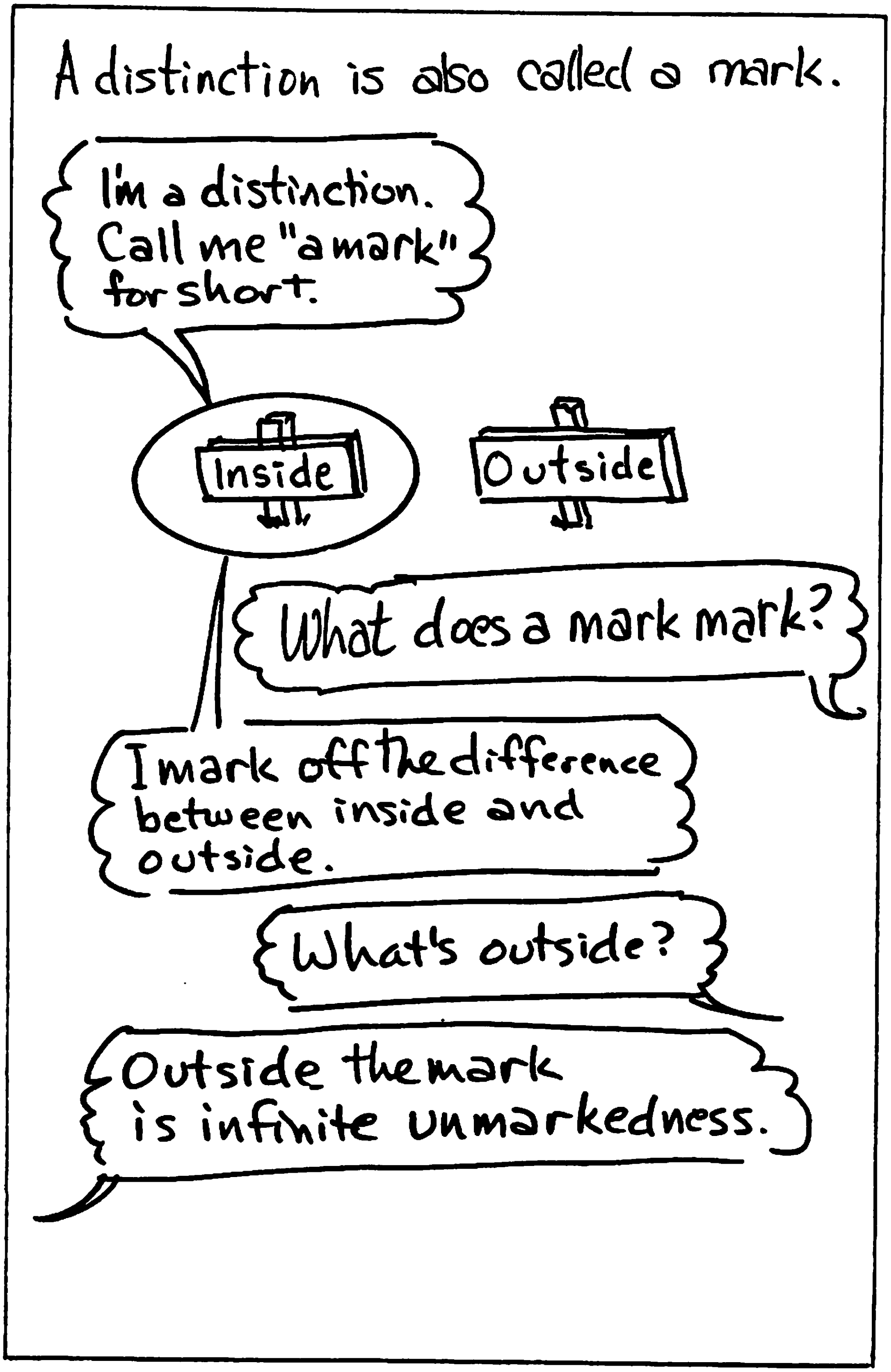
Heinz VonFoerster


But, a lot of people had a great deal of difficulty understanding what Spencer Brown was saying...
He introduces a new notation that feels like learning Russian! And the laws are stated so succintly. It's too dense for me.
Laws
of For
??
We hope this primer will help you understand Spencer Brown's ideas and their possible applications

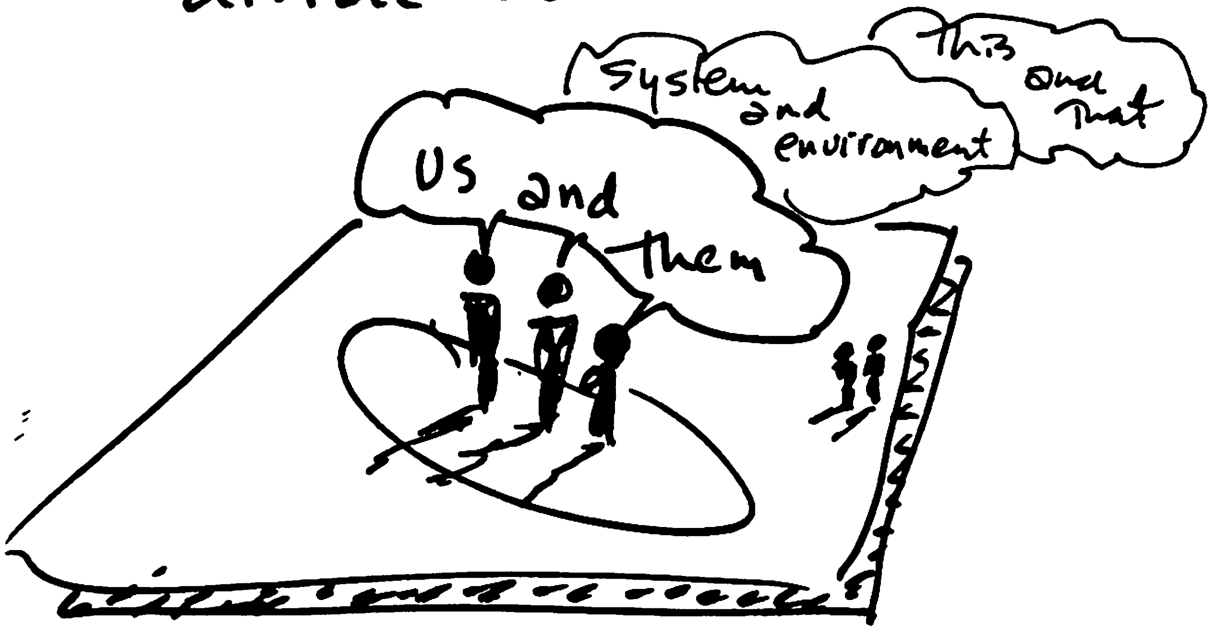

The first KEY IDEA in The Laws of Form is
A universe comes into being when a space is severed
Spencer Brown
what's an example of That?
I'm inside my skin. Everything else is outside!
So Spencer Brown starts his logic with The first injunction:
"Draw a distinction."
Draw a distinction? About what?
About anything... spaces, states, content... anyThing... The Laws of Form are about distinctions in general


How do you "draw a distinction?"
By arranging a boundary so you cannot reach a point on one side of a boundary from the other side without crossing the boundary.
That's it?
Yes, indeed
That seems simple enough. But what's an example?
A circle on a piece of paper is a distinction.
I'm a "distinction."


Do you actually have to draw the distinction on paper to "draw a distinction?"
Not at all. I could have expressed the first command in other ways.
Like what?
Let there be a distinction
or
Find a distinction
or
See a distinction
or
Describe a distinction
or
Define a distinction
or
Let a distinction be considered
O.K. O.K. I get the idea.


A distinction is also called a mark.
I'm a distinction. Call me "a mark" for short.
Inside
Outside
What does a mark mark?
I mark off the difference between inside and outside.
What's outside?
Outside the mark is infinite unmarkedness.

Note: By drawing distinctions we form concepts, and divide ourselves into



It's how we form our

- ☐ identities
- ☐ our social units
- ☐ our organizations
- ☐ our nations

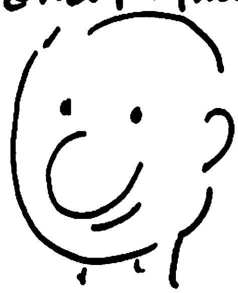
I'm Bob

This is The good ole USofA!

There are two types, or aspects, of distinctions...

1. those which we make intellectually (and emotionally and physically)

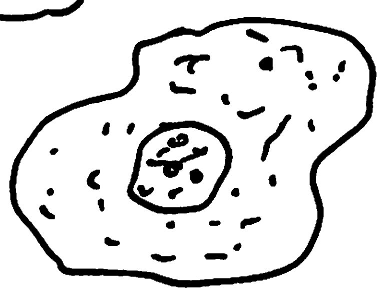
Example?



Hmmm... There is being and not being

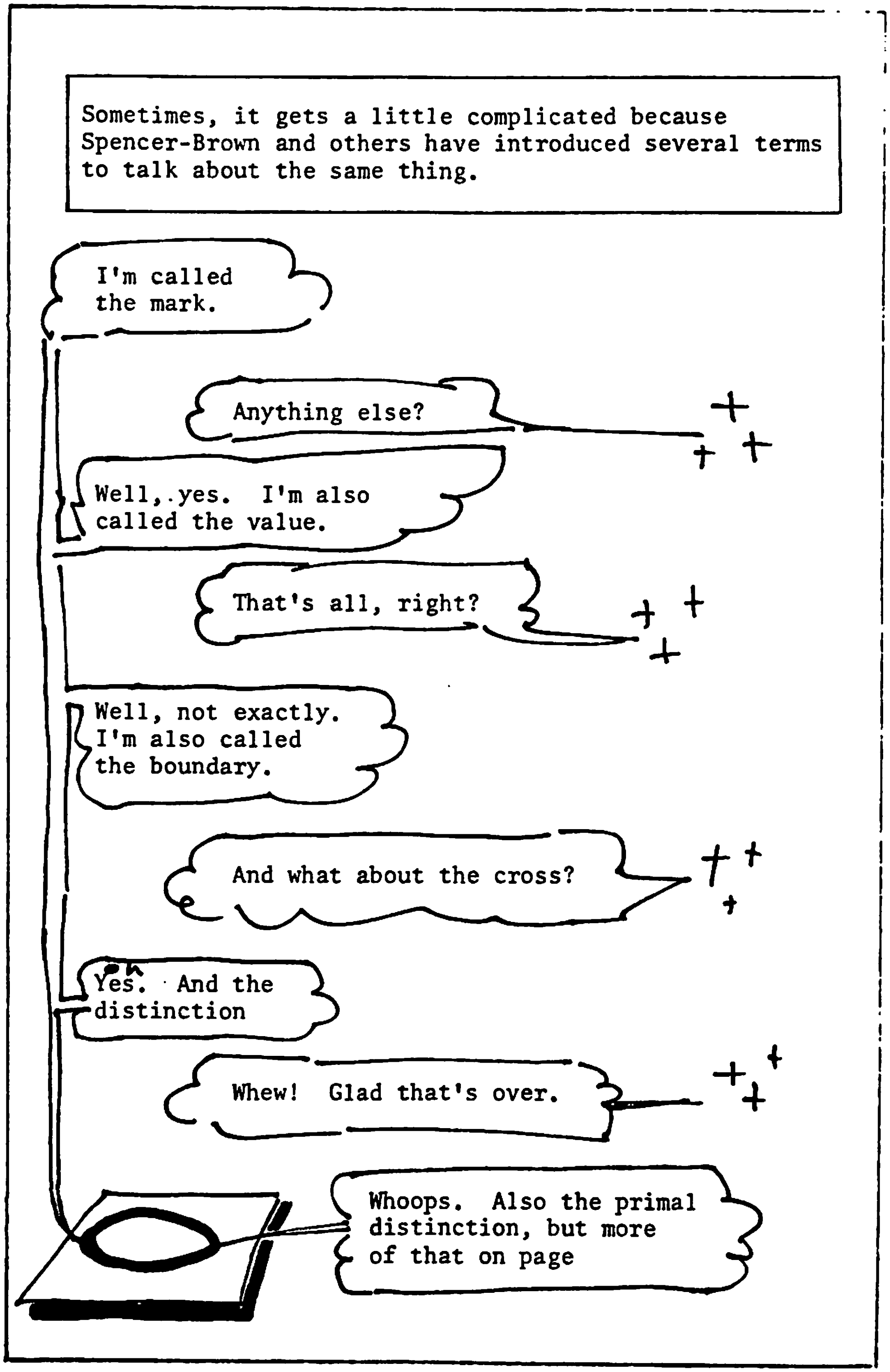
2. those which exist physically

Example?



The cell membrane makes the distinction (for the cell) of inside and outside

Sometimes, it's a little hard to determine just where the boundaries are... in fact it depends on your point of view...


Sometimes, it gets a little complicated because Spencer-Brown and others have introduced several terms to talk about the same thing.
I'm called the mark.
Anything else?
Well, yes. I'm also called the value.
That's all, right?
Well, not exactly. I'm also called the boundary.
And what about the cross?
Yes. And the distinction
Whew! Glad that's over.
Whoops. Also the primal distinction, but more of that on page

Why are the Laws of Form Important?

It is important to trace the way we represent such a severence. It allows us to reconstruct the basic forms which underlie linguistic, mathematic, physical and biological science.

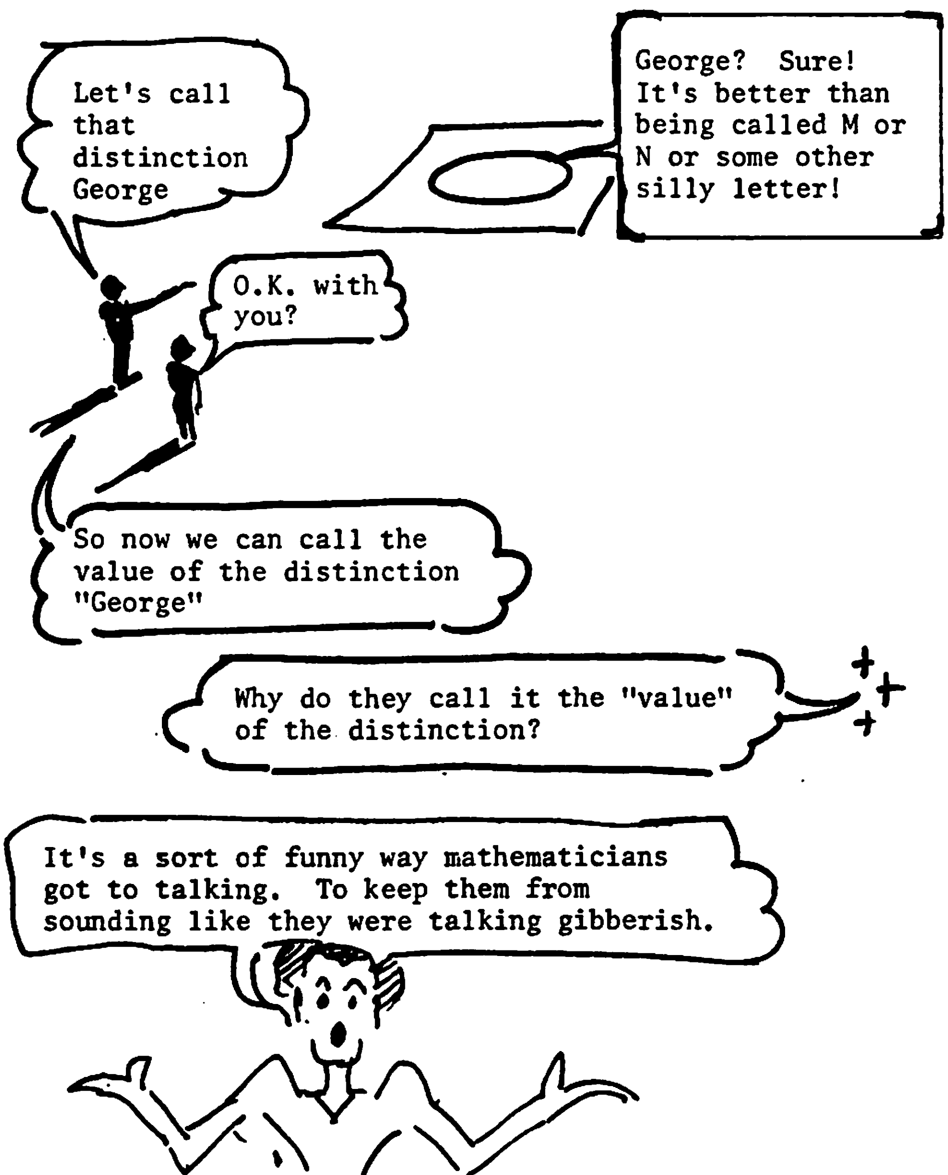
We can begin to see how the familiar laws of our own experience follow inexorably from the original act of severence. It becomes evident that the laws relating such form are the same in any universe (independent of how the universe actually appears!)

Spencer Brown

Sometimes people speak of the

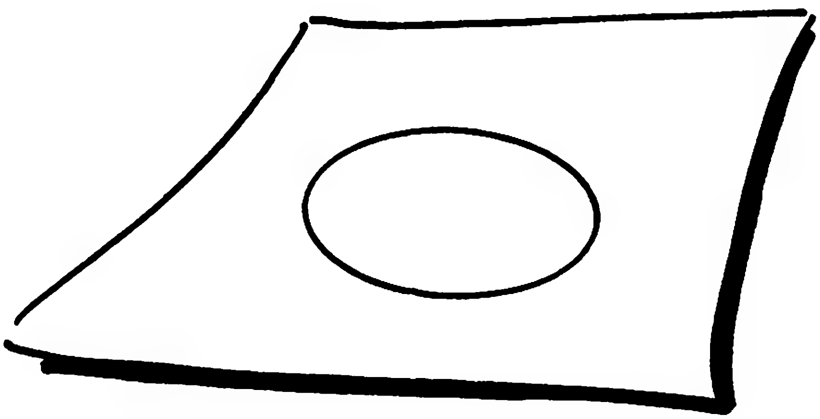
"value of a distinction."

What they mean is nothing more
than that they have given
the distinction a name.



Marks are used to distinguish.

For example, a circle distinguishes a certain type of space.



Any mark in any space is a distinction.

or draws a...

Spencer Brown certainly starts with simple stuff. I wonder why people find it so difficult?

Let's go through this again.
Why is understanding the
Laws of Form important?

Well for one thing, making distinctions
is fundamental to how you develop
descriptions.

You might call it the
Basis for a Theory of
Descriptions?

Yes. And don't forget
that this is what
science is ...a prescription for certain kinds of descriptions

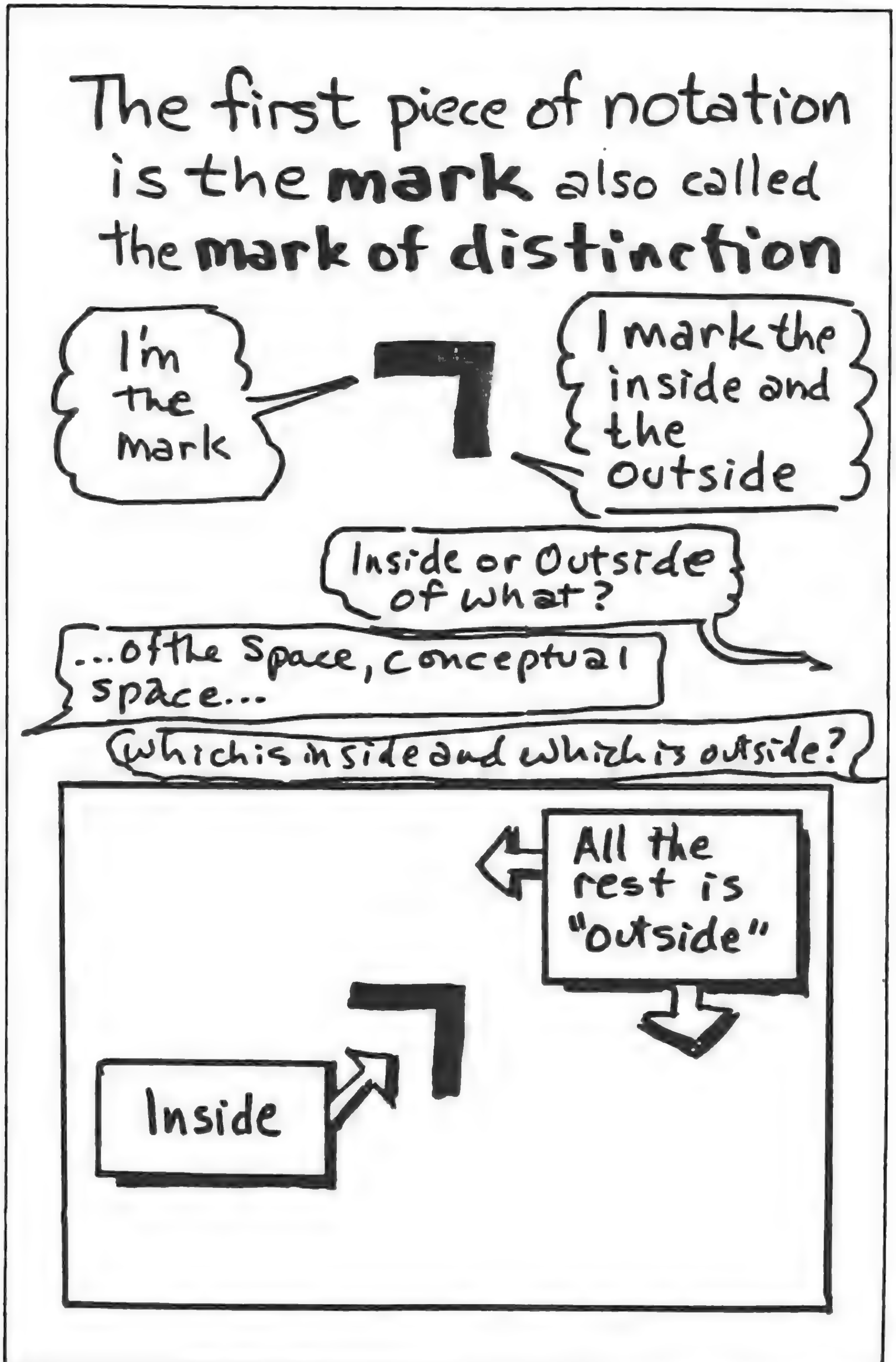
But, why go to the trouble of
studying this with symbols
and theorems and axioms and
all that stuff?

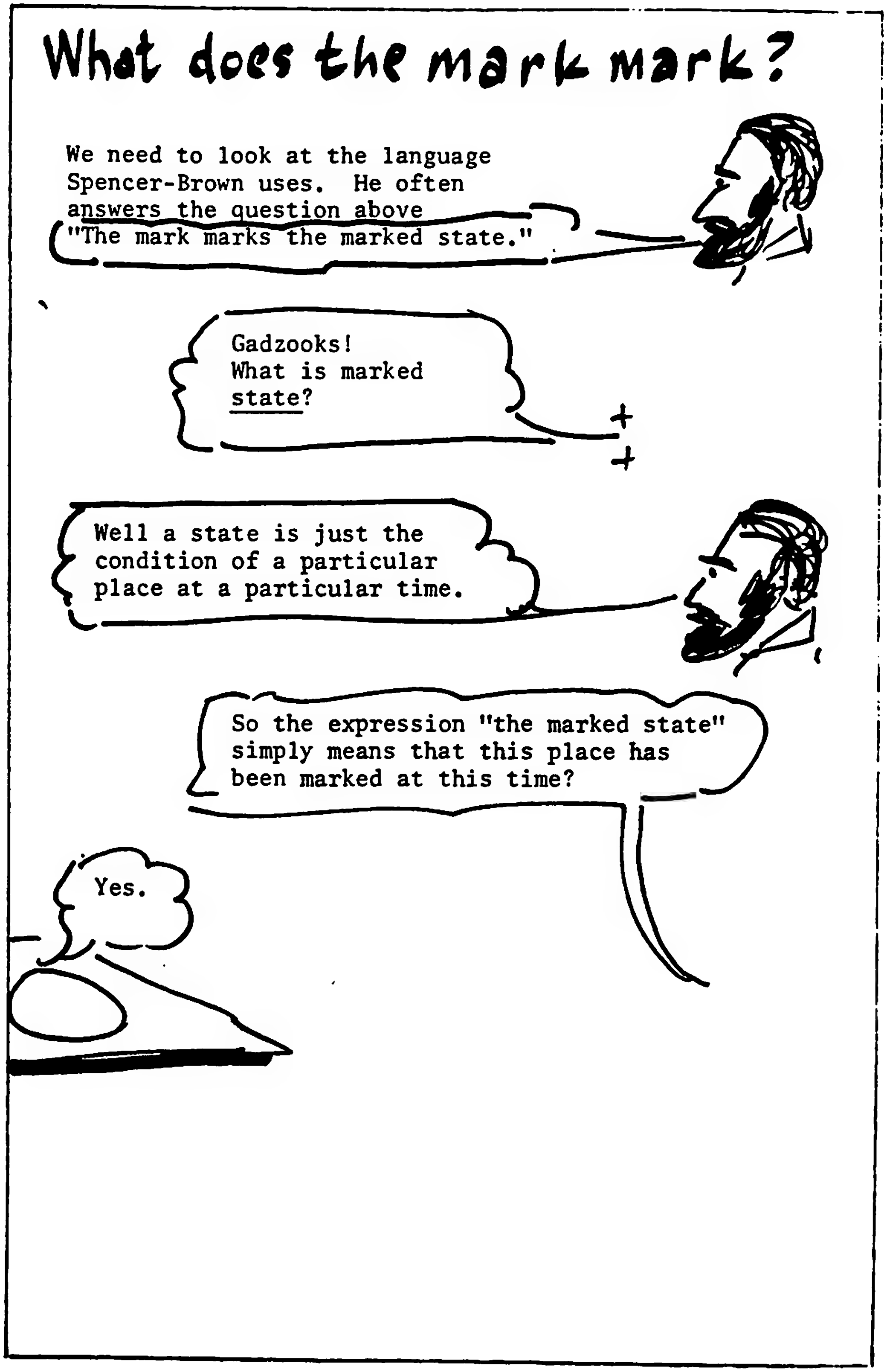
"The search for a rigorous formulation . . . has a more serious motivation than mere concern for logical niceties or the desire to purify well established methods of . . . analysis. Precisely constructed models . . . can play an important role, both negative and positive, in the process of discovery itself. By pushing a precise but inadequate formulation to an unacceptable conclusion, we often expose the exact source of this inadequacy, and consequently, gain a deeper understanding. . ." (Chomsky, 1957:5).

Chomsky

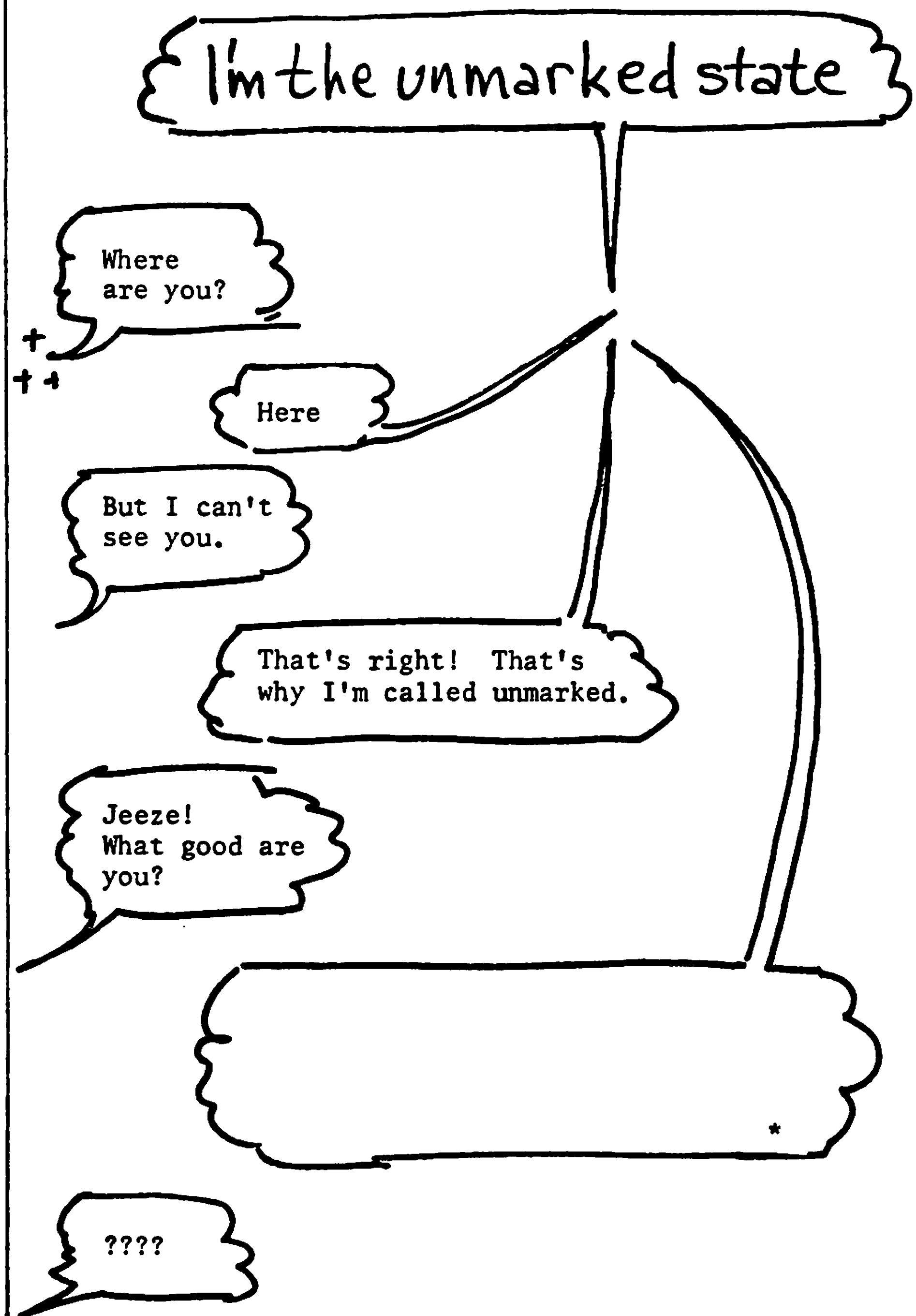
Chapter 2

The Notation and Axioms


The first piece of notation is the **mark** also called the **mark of distinction**
I'm the mark
I mark the inside and the outside
Inside or Outside of what?
...of the Space, conceptual space...
Which is inside and which is outside?
All the rest is "outside"
Inside


What does the mark mark?
We need to look at the language Spencer-Brown uses. He often answers the question above "The mark marks the marked state."
Gadzooks! What is marked state?
Well a state is just the condition of a particular place at a particular time.
So the expression "the marked state" simply means that this place has been marked at this time?
Yes.

The second piece of notation
is the unmarked state.



* indicates that there are notes at ~~the end of the book about this~~ topic.

The unmarked state is a little
hard to see sometimes

Sometimes?!?!
Try and find
me right now!

So, we introduce quote marks
to indicate the unmarked state.

“ ”

Here I am!

That makes it
easier to
see me,
doesn't it?

*

The third symbol we introduce
is the familiar equals sign.

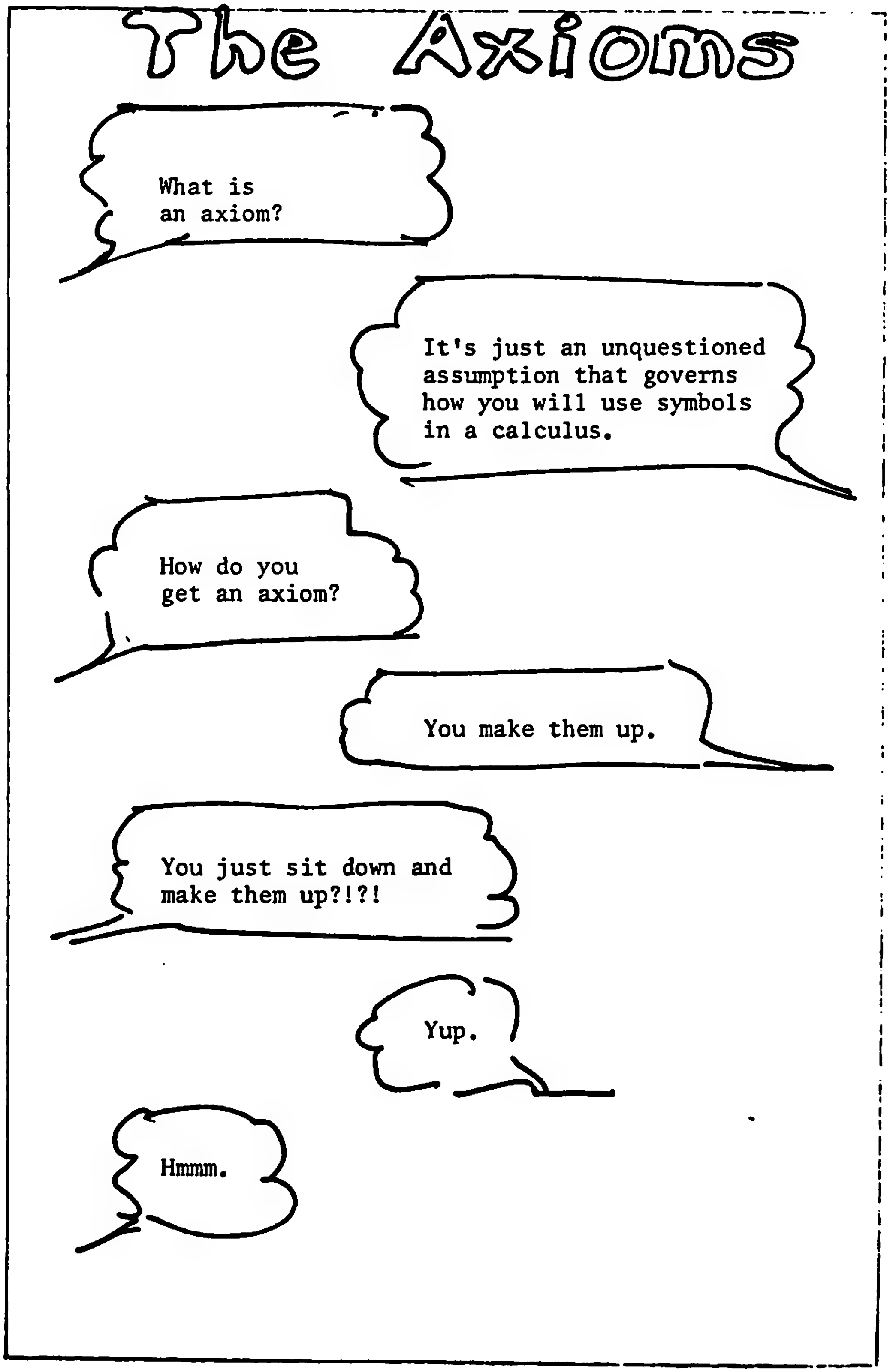
$$=$$

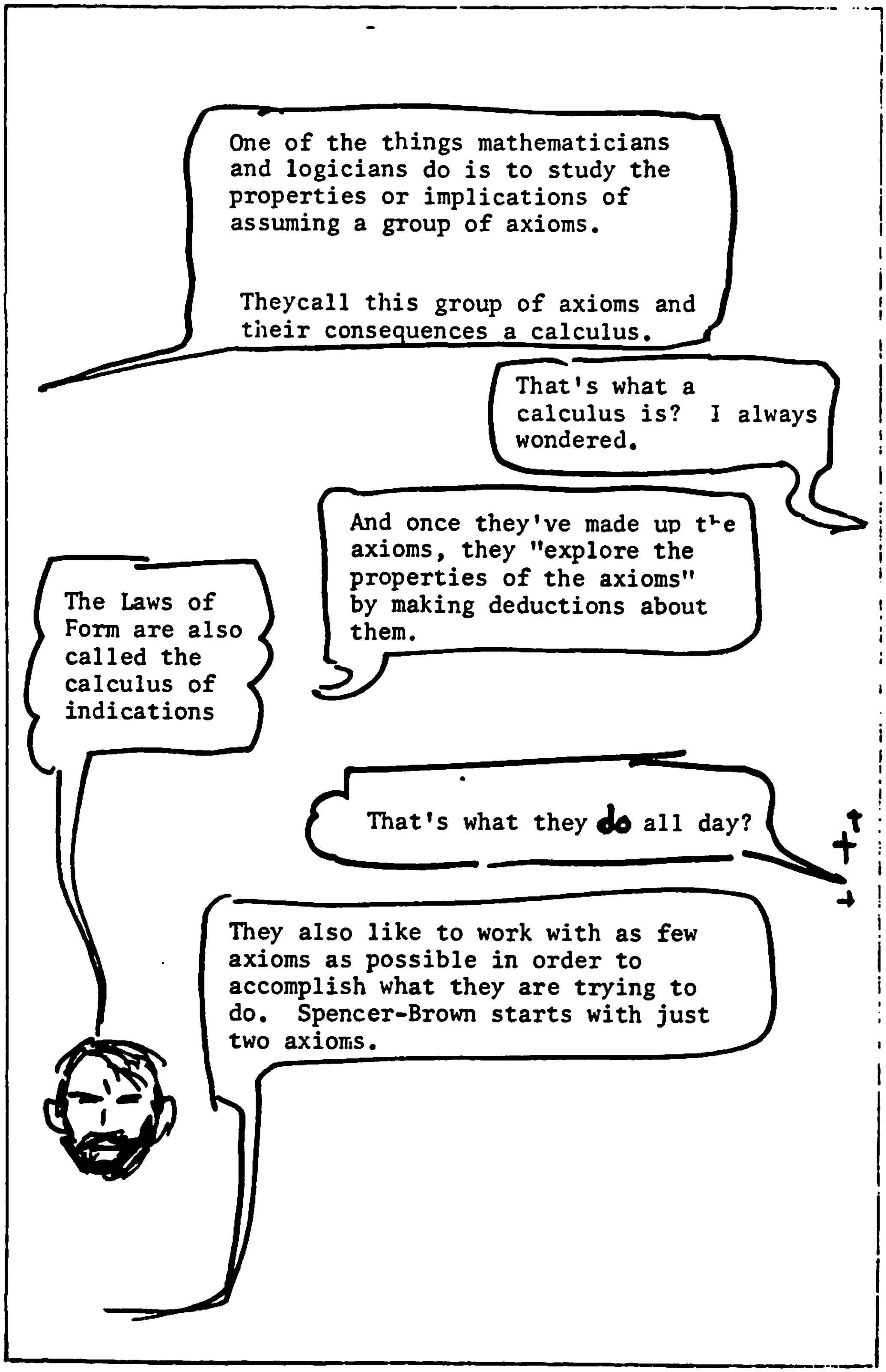
It means (or is translated
into English as)

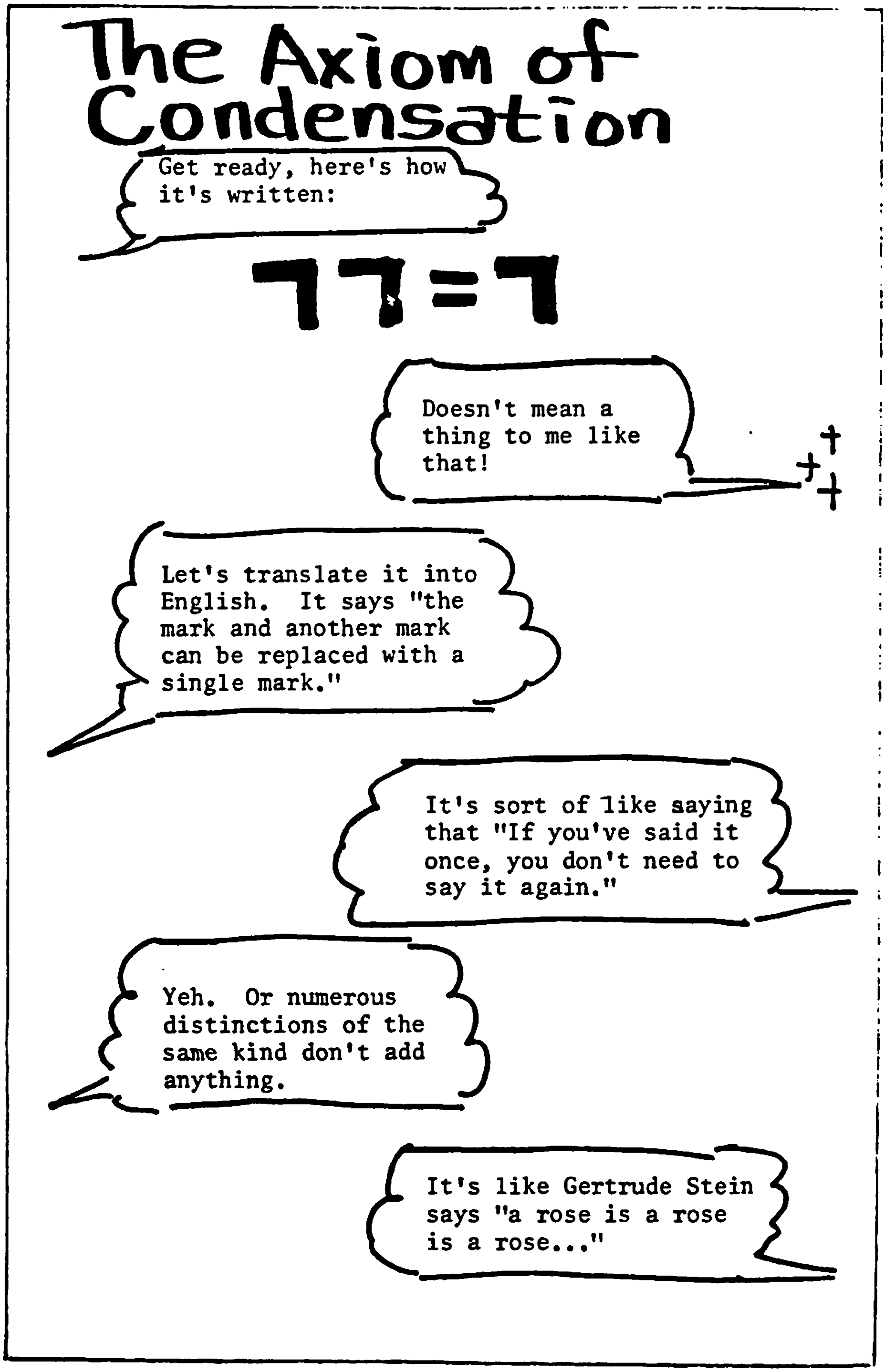
can be replaced with

You may also want to use the term "is equivalent to".

Now, we are ready to start getting acquainted with the Axioms.

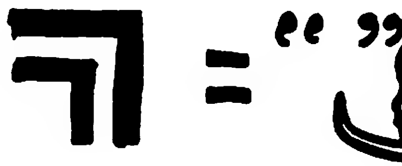

The Axioms
What is an axiom?
It's just an unquestioned assumption that governs how you will use symbols in a calculus.
How do you get an axiom?
You make them up.
You just sit down and make them up?!?!
Yup.
Hmmm.


One of the things mathematicians and logicians do is to study the properties or implications of assuming a group of axioms.
Theycall this group of axioms and their consequences a calculus.
That's what a calculus is? I always wondered.
And once they've made up the axioms, they "explore the properties of the axioms" by making deductions about them.
The Laws of Form are also called the calculus of indications
That's what they do all day?
They also like to work with as few axioms as possible in order to accomplish what they are trying to do. Spencer-Brown starts with just two axioms.


The Axiom of Condensation
Get ready, here's how it's written:
Doesn't mean a thing to me like that!
Let's translate it into English. It says "the mark and another mark can be replaced with a single mark."
It's sort of like saying that "If you've said it once, you don't need to say it again."
Yeh. Or numerous distinctions of the same kind don't add anything.
It's like Gertrude Stein says "a rose is a rose is a rose..."

The Axiom of Cancellation

This looks even stranger than the first one.



I'm here but not seen, of course.

Here's its translation into English: "One mark inside another mark can be replaced with no mark."

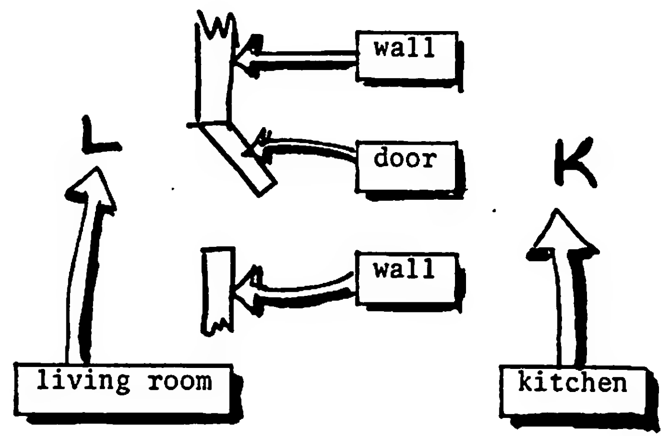
I don't get it.

Perhaps an illustration will help.

I'll try anything once.

We'll use the Go Through the Door example.

Suppose there are 2 rooms and a door which we can diagram this way



We start on one side of the door, let's say the Living Room side.

Now let us say that the sign ⌉ stands for the command: "go through the door."

OK so far

Suppose we write

⌉⌉

It translates "Go through the door; go through the door."

Exactly.
so why do
you say it
at all?

So why say it twice? That's exactly what the Axiom of Condensation is saying (only more succinctly and in symbols)

$$\underbrace{\neg}_{\text{Go through the door}}\ \underbrace{\neg}_{\text{Go through the door}}\ \underbrace{=}_{\text{Can be rewritten as}}\ \underbrace{\neg}_{\text{Go through the door}}$$

Here it is on the illustration

Go through the door Go through the door.

That makes sense. What about the other Axiom?

The Axiom of Cancellation is a little different.

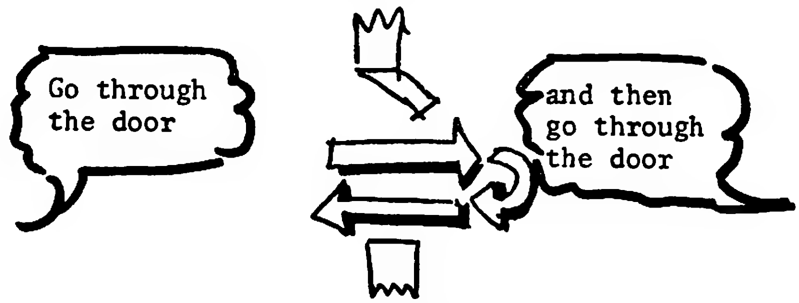
It says $\underbrace{\neg\!\!\mid = \text{“ ”}}$

Go through the door

AND THEN

Go through the door.

Note that saying this is different. Here's the illustration.



This equivalent to not having gone through the door at all.

Right! Because you are really right back where you started.

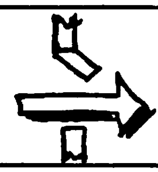
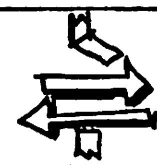
So if you start in the unmarked state " " and receive the command you would (go through the door) to the marked state; and then when you would receive the command (go through the door) you would go to the unmarked state--which is the equivalent of not having received it at all.

So look at the Axiom of Cancellation again.

$$\urcorner\!\!\!\urcorner = \text{“ ”}$$

OK - I think I understand.

Here's a $\boxed{\text{summary.}}$

AXIOM OF CONDENSATION	AXIOM OF CANCELLATION
Go through the door; go through the door	Go through the door; and then go through the door
	
$\urcorner\urcorner = \urcorner$	$\urcorner\!\!\!\urcorner = $ “ ”

If you name (or mark) something twice, actually all you've done is to name it once. If you name it and then unname it, you're back at the same place you started where you hadn't named it.

Next we'll try the Circle Example

We've said that the mark indicates one side from another, in a sense it marks the inside from the outside.

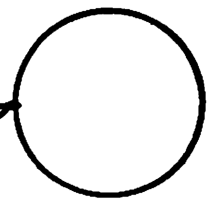
OK so far

So, the mark doesn't have to be written like this:

$\neg$

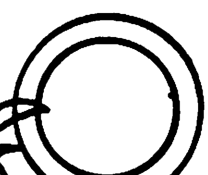
It could be written as a circle

I indicate: what's inside is not outside; and what's outside is not inside.



Sure! A circle is a mark in a space.

So now we write the Axiom of Cancellation as one circle inside another



= " "

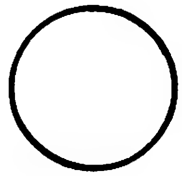
Whoops! Can you translate that?

It's not inside and it's not outside either.

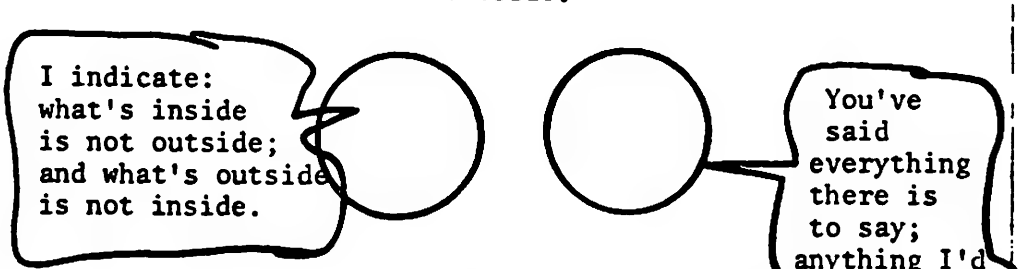
OR

If the outside and inside are marked by the same distinction, then there is no distinction about inside or outside.

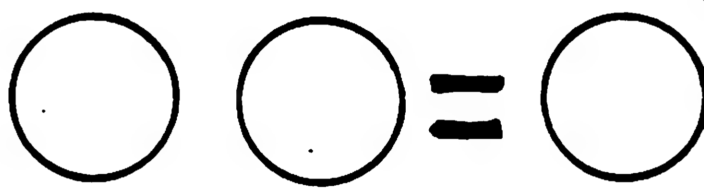
Let's look at this "inside/outside" language. Make a mark



Note that it marks the inside and the outside. Now put another mark to mark the outside.



But, the outside is already marked by the first mark, so...



Two marks of the outside are equivalent to one mark of the outside (and inside). Why mark it more than once?

Chapter 3

Looking at Expressions

Expressions

Expressions are groups of marks

and nomarks! Don't forget me

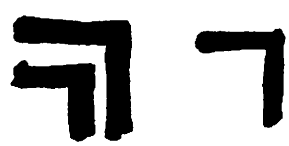
on either side of an equals sign

Why do it that way?

Because a lot of mathematics consists of exploring what is equivalent to what in the domains described by a set of axioms

Unclear

So an example of an expression might be



In this calculus, all of what you are doing is trying to simplify expressions to see if they end up in the marked state or the unmarked state.

That's all?

That's the basics of it (as we'll see in Chapter There are some pretty interesting results when you start using a symbol for self reference.)

Self reference?

Yes, but don't think about that now.

Evaluating an Expression

First, you have to have an
expression such as

$$\overline{\overline{\quad}\big|}\Big|\ \ \overline{\quad}\Big| = ?$$

What you want to find out is whether the "value" of this expression is the marked or the unmarked state.

How to do it?

We apply the Axioms of Condensation and Cancellation to the expression until we get a single state, either a marked or unmarked one.

How do we do that?

We look at an expression and ask ourselves "how can I apply one of the two axioms--condensation or cancellation-- to it?"

Could I have an example?

Here's an example:

Suppose we start with this expression:

and we want to know how to simplify it to the marked or unmarked state.

What we can do is to apply the Axiom of Cancellation to it.

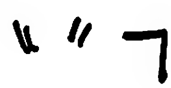
What was the Axiom of Cancellation?

It is

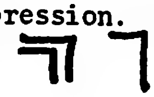
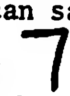
Oh, right. Now I remember.

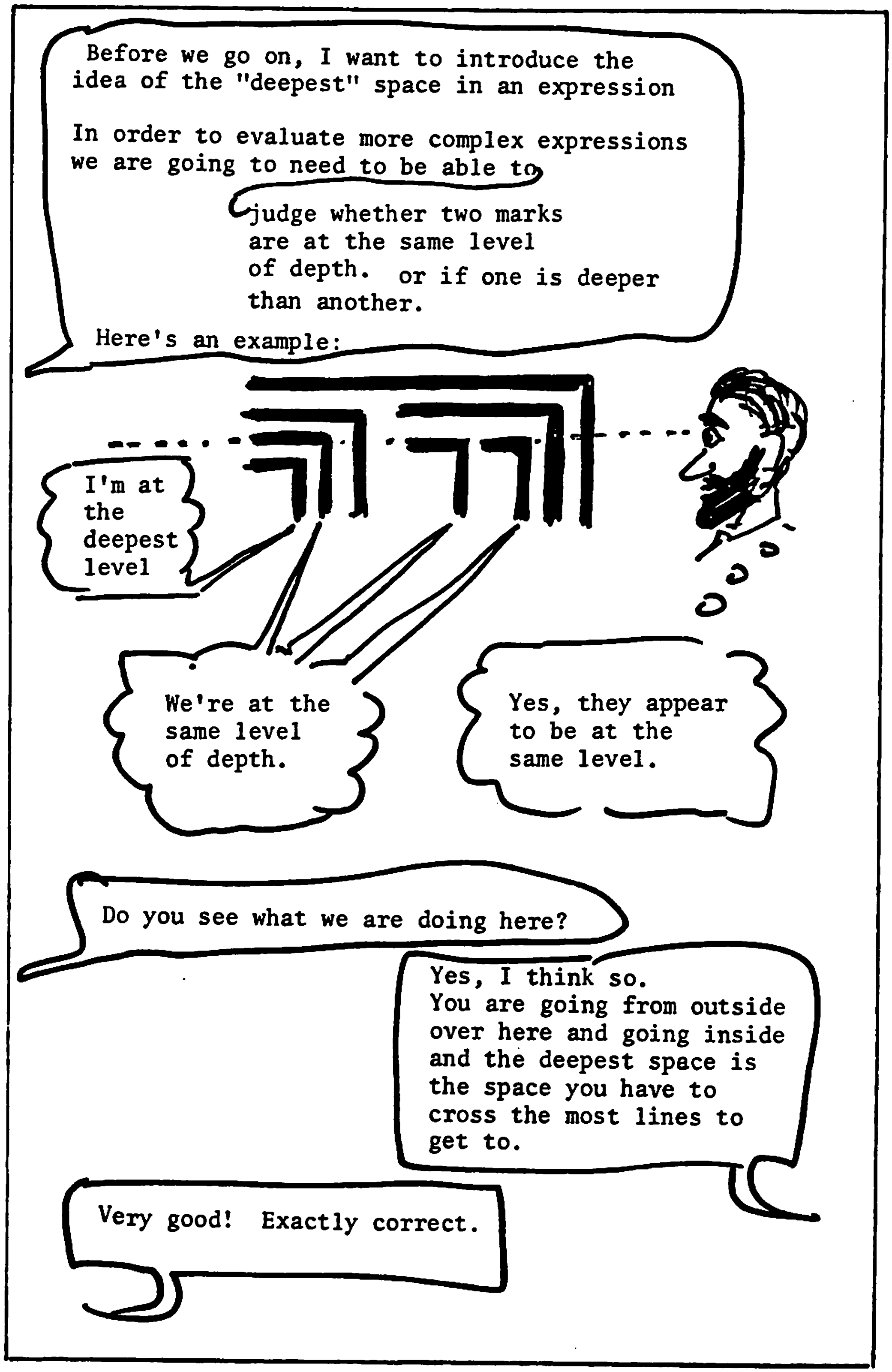
So we can then apply it to part of the expression, right here.

So, we then write



I see. Because you put in the unmarked state for the two marked states, one inside the other.

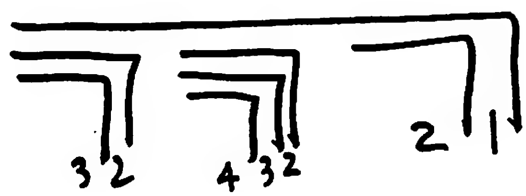
Right. And since all you have left is the marked state, and you can't apply either axiom again to that, you are finsihed with evaluating this expression. We can say the value of  is 


Before we go on, I want to introduce the idea of the "deepest" space in an expression
In order to evaluate more complex expressions we are going to need to be able to
judge whether two marks are at the same level of depth. or if one is deeper than another.
Here's an example:
I'm at the deepest level
We're at the same level of depth.
Yes, they appear to be at the same level.
Do you see what we are doing here?
Yes, I think so. You are going from outside over here and going inside and the deepest space is the space you have to cross the most lines to get to.
Very good! Exactly correct.

You can actually count the spaces

And you can even mark the levels
with numbers if it is hard to
see which level they are.

Look at what we did with this expression,



I think I get it .

You just put a number in the space
depending on how many lines you cross
from outside.

Example 2 - Evaluating Expressions

Shall we do a harder one?
Try this

The question as always
here is "Does it
reduce to the
marked state or
the unmarked state?"

It has two "deepest" spaces.
First we apply the Axiom of
Cancellation to the left-
most of the spaces and get

" " 

Then we apply the Axiom of
Cancellation to the other
deepest space

and we get " "

Again we have reached the
stopping place. We have
evaluated the expression

 = ?

and found it has the value
of the unmarked state.

I noticed you said "left-most"
when telling which one to start
with. Do you always start with
the left-most deepest space.

Yes. That is a good rule
to follow in simplification.

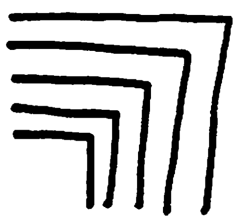
There is one more thing
we can do with the mark,
we can draw it to
cover a whole set of
other marks.

The marks inside are regarded
as deeper than the one that
are drawn over them.

That means that you could
have all sorts of
strangelooking expressions.

Such as?

That's pretty strange.

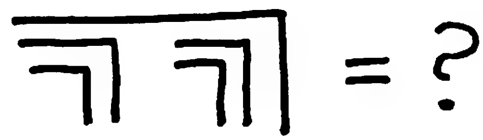


That is very strange.

And deep.

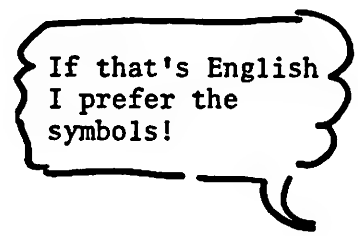
Here's a couple of expressions
and how they are translated
into English

Expression
A



Translation
for A

The expression mark inside
mark followed by a mark
inside a mark all of these
marks inside a mark can be
transformed into what?



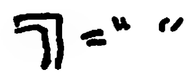
Expression
B

Translation
of B

The expression mark inside
mark inside mark is equiva-
lent to what?

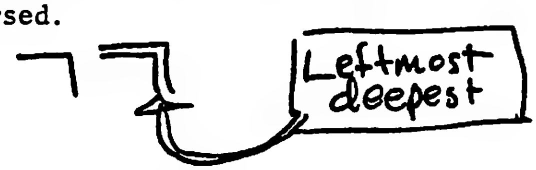
Start your simplification at the
left-most, deepest mark.

Apply the cancellation axiom.

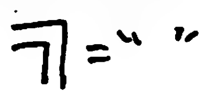


That leaves only the mark.

Try another one. Here's the same
one reversed.



Apply the cancellation axiom.

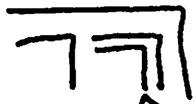


And you end up with the same thing.

I think I have that
so far.

Then let's try a slightly
harder one.

Try this one:

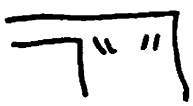


The left-most deepest
is here.

Right .

Now we apply the
cancellation axiom to it
and the form around it.

and we get



which is the same as

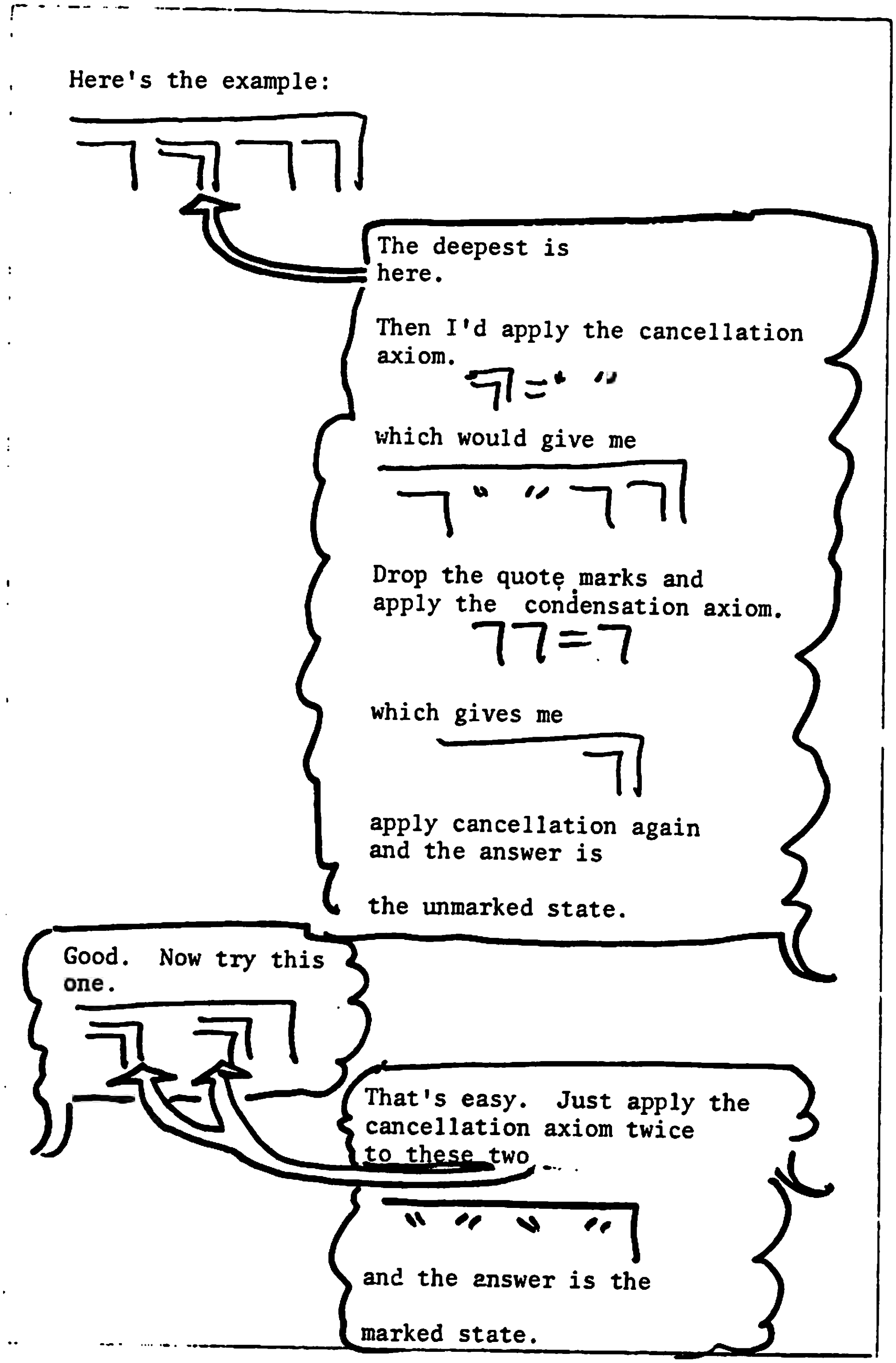
which if we apply the
cancellation axiom once
again we get

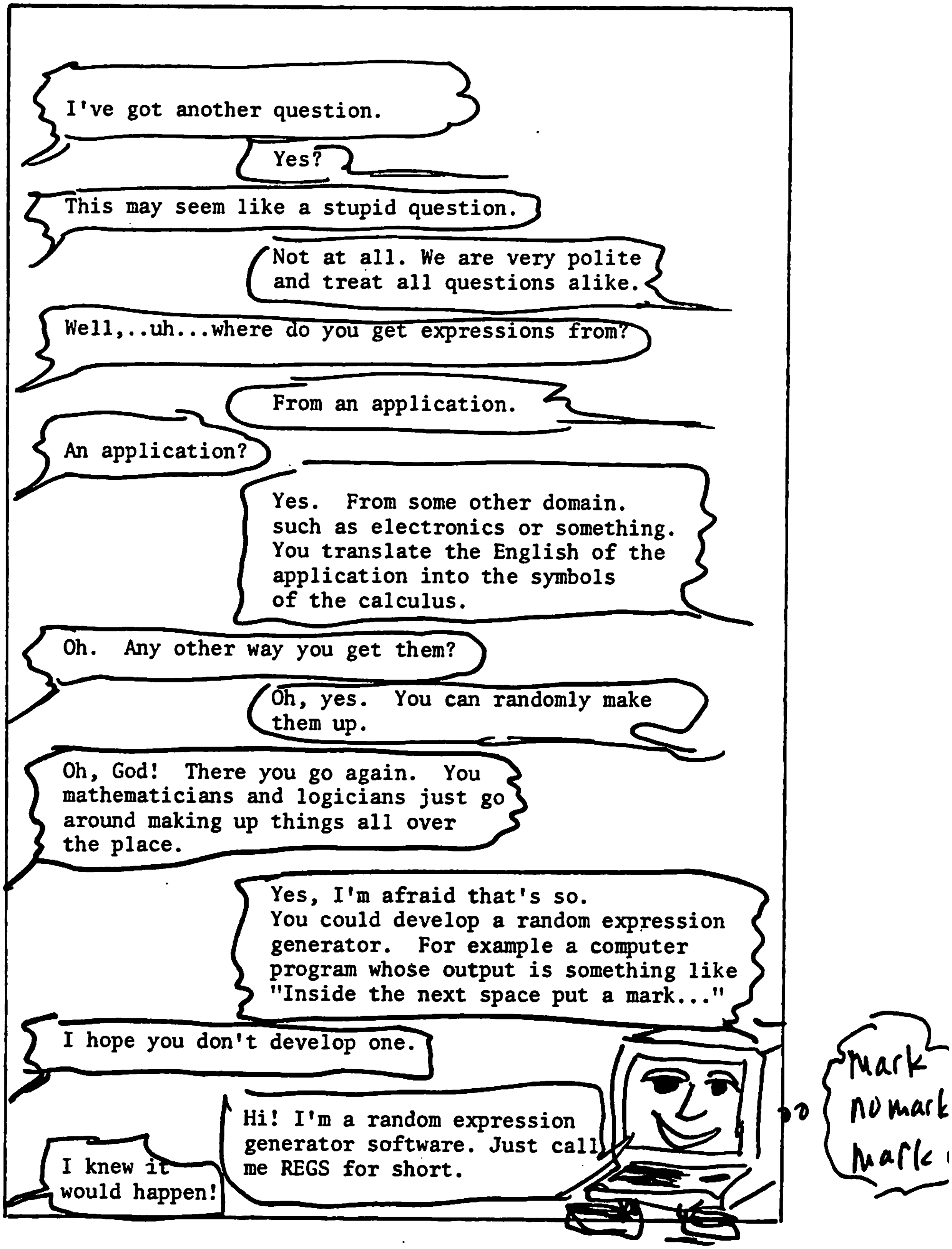
"   "

Here I
am, still unmarked

Let's try a slightly
more complicated one.

O.k. But let me
try doing it.

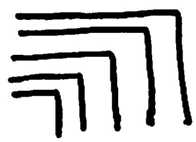

Here's the example:
The deepest is here.
Then I'd apply the cancellation axiom.
which would give me
Drop the quote marks and apply the condensation axiom.
which gives me
apply cancellation again and the answer is
the unmarked state.
Good. Now try this one.
That's easy. Just apply the cancellation axiom twice to these two ...
and the answer is the
marked state.

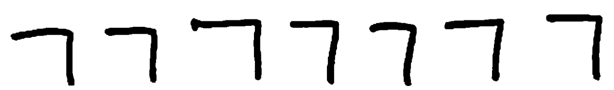

I've got another question.
Yes?
This may seem like a stupid question.
Not at all. We are very polite and treat all questions alike.
Well,..uh...where do you get expressions from?
From an application.
An application?
Yes. From some other domain. such as electronics or something. You translate the English of the application into the symbols of the calculus.
Oh. Any other way you get them?
Oh, yes. You can randomly make them up.
Oh, God! There you go again. You mathematicians and logicians just go around making up things all over the place.
Yes, I'm afraid that's so. You could develop a random expression generator. For example a computer program whose output is something like "Inside the next space put a mark..."
I hope you don't develop one.
Hi! I'm a random expression generator software. Just call me REGS for short.
I knew it would happen!
Mark
no mark
Mark

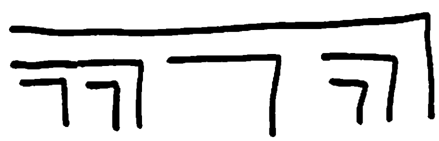
Exercises

In math, you usually have to do a few examples to really feel comfortable with them. Try your hand at these:

1.  =

2.  =

3.  =

4.  =

Where are the answers?

Look on page

Chapter 4

A Brief Introduction to the Algebra

or how to use The shuffling rules

One of the things that Spencer Brown does is to develop an algebra based on the indications.

Oh, oh. I'm getting out of here. That just means more notation.

Well, stick around. We'll try to make is as simple as possible.

O.K. I'll stay for a while. But algebra is awefully abstract.

Basically, all you are doing is learning how to shuffle around the symbols in this calculus.

But why do you want to shuffle them around at all?

The main goals are to be able to simplify the expressions, no matter how complicated, to see whether they come out to be the marked or unmarked state.

And why do you want to be able to do that?

That's a bit harder to answer. When we do get to applications, you want to be able to simplify all of these complicated expressions to solve problems. For example, Spencer Brown applied the indicational calculus to electronic circuit design.

I'm never going to design circuits.

No. Neither am I. But you may have some logical problems in your field that the indicational calculus may help you with.

O.K. so let's get on with the algebra.

Some more notation

Before we go on with the main rules of the algebra, we want to introduce some more notation.

We can let a letter stand for an expression. We call that letter a variable. Remember it can stand for either a marked or an unmarked state.
So we could write

$$a = \overline{\overline{}|}|\ \ \overline{}|$$

Which is read "a is an expression that is equivalent to mark inside mark followed by mark."
So, when we use a letter it means

"an expression
or
some expression
containing
some combination of marks."

With this notation you can write such expressions as

$$a = b$$

which can be translated into English as the expression **a** is equivalent to the expression **b**.

Yeh, but what about the marks?

Well, it means that when the marks that **a** stands for are simplified are the marks which **b** stands for are evaluated; you will end up with the same value for **a** and **b**.

Which is either the marked or unmarked state?

Yes.

Now, you can also write an expression like this

$$e = \overline{f\,}\rceil$$

which can be translated...

"The expression e is equivalent to some expression inside a mark"

OR it can be translated...

"The expression e can be transformed into a marked state or a non marked state inside a marked state."

Do you know what f is?

No--f can be either marked or unmarked. It's a variable.

And a variable is...?

A variable is a letter we use to mark a state where we don't (for the moment at least) know whether it is marked or unmarked.
And remember these are the only two possibilities.

Substitution

You can substitute either the marked state or the unmarked state for a letter.

You can? Either one?

Sure. Remember, an expression is going to be simplified into either a marked or an unmarked state.

Right.

So, what the letter does is to enable you to be vague for a while about whether it is a marked state or an unmarked state.

For a while?

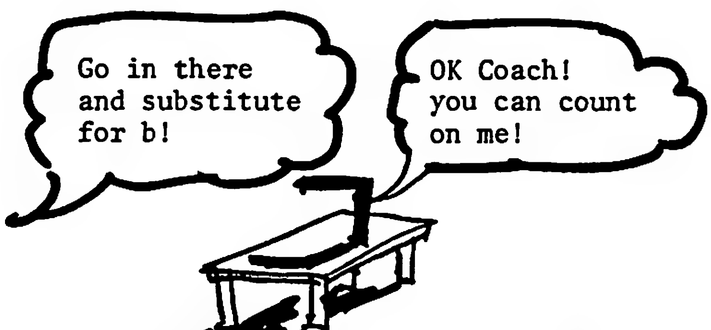
Yes, until you actually substitute either the ⌉ or the " " or a whole string of them into expression.

Let's look at one example.

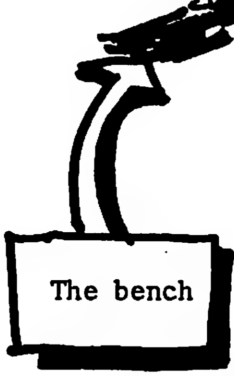
Here's an expression

a = b⌉

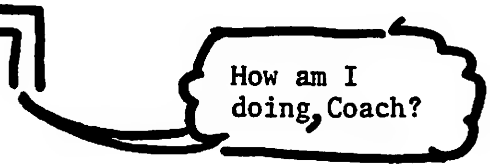
Now let's make a substitution.
Let's make b a marked state.



Here is the expression
after substitution.

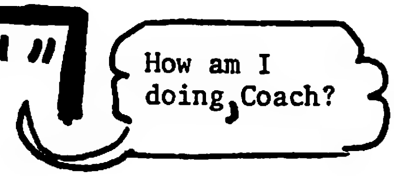


a = ⌉



We could also substitute in the
unmarked state. Here is the
expression a = b⌉ after
substitution of " ".

a = " "⌉



You see then you can simplify
the expression, as we did
in the previous chapter.

So, let's review.

You take the expression

$$a = \overline{b\,|}$$

Substitute the marked state for
b and you get

$$a = \overline{\overline{\;|}\,|}$$

And to simplify it you
use the good old axiom
of cancellation.

Right! and you get

a = " "

And of course you can remove
the quote marks which are just
there to remind you of the unmarked
state, so you end up with

a =

Yes. I see how you do it.
But every time I see that
equals sign with nothing
on the other side, it makes
me uncomfortable. I keep
wanting to put a symbol in
there of some sort. I feel
a compulsion to symbolize
nothing.

You could do it, as we have done
with the quote marks, but Spencer
Brown doesn't.

Look at what happens when you substitute in the unmarked state for b in the same expression

$$a = \overline{b\,}|$$

You put in the unmarked state for b and you get

$$a = \overline{"\;\;"}|$$

That's right and you can eleminate the quote marks and end up with

$$a = \overline{\quad}|$$

One other thing. You can do this with much larger sets of expressions as well.

Such as?

Such as

$$a = \overline{\overline{b\,}|\;\;\overline{\overline{b\,}|}|\;\;b\,}|$$

or

$$a = \overline{\overline{b\,}|\;\;\overline{\overline{c\,}|}|\;\;\overline{\overline{\overline{d\,}|}|}|}|$$

You can aay, let's suppose that b is always the unmarked state or the marked state and try it out.

Or you can say let's suppose that b, c, and d are all the marked or unmarked state, or that b is marked and c and d are unmarked.

Why would I want to do that?

To see what patterns result. We'll have more of that later.

Initials of the Algebra

Perhaps the best way of proceeding is just to show you the different rules of shuffling around the notation.

O.K. Show me the shuffling rules.

First of all there is one that looks like this

$$\overline{p\,\rceil\, p\,}\rceil \;=$$

I'm here

Spencer Brown calls this the position initial.

Wait a minute! What's an initial?

An initial is just a rule that you start with initially.

You mean he doesn't prove it?

Not in the algebra. He has proved it for what he calls the arithmetic of the indicational calculus. So he feels that he can take it as an initial.

What do you do with it?

Well if you have an expression that looks like this

$$\overline{a\,\rceil\, a\,}\rceil$$

you know immediately that it is equal to the unmarked state.

By just substituting p for a?

That's right.

All right. Let's go on.

He has another initial. He calls it

transposition

And it looks like this:

$$\overline{\overline{pr|}\;\overline{qr|}\,\Big|} = \overline{\overline{p|}\;\overline{q|}\,\Big|}\;r$$

> Whew! That one is a little longer. What can you do with it?

Just like the other one. If you have an expression that looks like it but with different letters, you can substitute and shuffle these letters around.

> Why would you want to do that?

In order to be in a better position to use other rules to finally simplify the whole expression.

> O.K. Then what?

Well, he proceeds to identify a whole group of what he calls

consequences

> What are they?

They are particular patterns that can be proved to always happen by shuffling around the initials.

They are all listed on the next page.

Here are the consequences and their names an abbreviations

Number	Consequence	Name	Abbreviation
C1	$\overline{\overline{a}\rvert}\rvert = a$	Reflexion	ref
C2	$\overline{ab}\rvert\, b = \overline{a}\rvert\, b$	Generation	gen
C3	$\overline{\ \ }\rvert\, a = \overline{\ \ }\rvert$	Integration	int
C4	$\overline{\overline{a}\rvert\, b}\rvert\, a = a$	Occultation	occ
C5	$aa = a$	Iteration	ite
C6	$\overline{\overline{a}\rvert\,\overline{b}\rvert}\rvert\,\overline{\overline{a}\rvert\, b}\rvert = a$	Extension	ext
C7	$\overline{\overline{\overline{a}\rvert\, b}\rvert\, c}\rvert = \overline{ac}\rvert\,\overline{\overline{b}\rvert\, c}\rvert$	Echelon	ech
C8	$\overline{\overline{a}\rvert\,\overline{br}\rvert\,\overline{cr}\rvert}\rvert = \overline{\overline{a}\rvert\,\overline{b}\rvert\,\overline{c}\rvert}\rvert\,\overline{\overline{a}\rvert\,\overline{r}\rvert}\rvert$	Modified Transposition	mod
C9	$\overline{\overline{\overline{b}\rvert\,\overline{r}\rvert}\rvert\,\overline{\overline{a}\rvert\,\overline{r}\rvert}\rvert\,\overline{\overline{x}\rvert\, r}\rvert\,\overline{\overline{y}\rvert\, r}\rvert}\rvert = \overline{\overline{r}\rvert\, ab\, rxy}\rvert$	Crosstransposition	cro

Do I have to remember the names?

No, as a matter of fact, you don't have to remember the Consequence either. You can always look them up if you are doing a long example.

We should probably do some examples so that you can see just how this works. And then you'll be able to do them yourself.

Examples! I thought I wouldn't have to do anything in a primer.

Let's try a simple one.

Great! I like the simple ones!

Suppose you had the following example and wanted to simplify it.

You mean find out whether it represents the marked or unmarked state?

Yes. Here's how you'd go about it. You're going to suppose that r is either the marked or the unmarked state in turn.

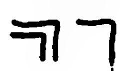
It has to be one or the other?

Yes, Spencer Brown proved that you can simplify expressions to one or the other.

O.K.

So first we'll try r =

That means that when we substitute the mark for r we get



Right.

Now, remember the rule for simplifying the expressions?

Sure, I remember the rule. Start your simplification at the left-most, deepest mark.

Right! So do that.

O.K.

This is the deepest mark. and I apply the axiom of cancellation.

which leaves me with

Drop the quote marks because they only indicate that I have an unmarked state here. And my answer is the marked state.

Good. Now substitute the unmarked state into that expression

Start with

Substitute the unmarked state twice here and here

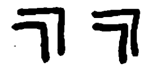
Drop the quote marks and my answer again is the marked state.

I have to admit that these are pretty easy.

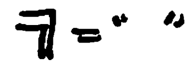
Let's try this one.

I'll do it. First I going to substitute in the marked state. Then later I'll substitute in the unmarked state.

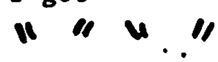
So, first the marked state.



Do the old cancellation axiom twice.



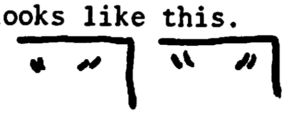
and I get



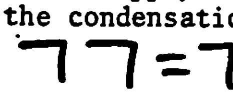
Eliminate the quotes and the answer is theunmarked state.

Then I try substituting the unmarked state in the original expression

This time ,after subsititution , it looks like this.



Eliminate the quotes and it looks like I should apply the other axiom, the condensation axiom.



So my answer is the marked state.

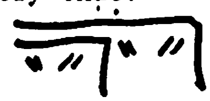
Hey, I got two different answers depending on whether I substituted the marked or unmarked state. Is that possible? Did I do it correct?

Yes, you did it correctly.
And yes, it is possible to get the two different answers depending on what you substitute for the variable.

So, what we can conclude from this is that at least in this instance some expressions cannot be simplified any further when they contain variables.

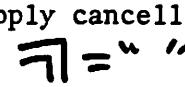
Let's try one more. And see how it comes out.

First let's substitute the unmarked state and try that.



Eliminate the quotes.
That gives me

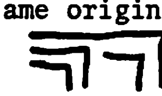
Apply cancellation.



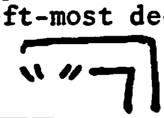
Drop the quotes and the answer is the unmarked state.

Good.

Now substitute the marked state into the same original example.



Apply cancellation to the left-most deepest position.

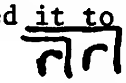


Then I apply cancellation again and I get the unmarked state.

I got the unmarked state twice.

Hey, wait a minute. This example I just did looks alot like one of those initials.

Yes, it is one of the initials. I slipped it in on you. But now you have proved it to yourself that



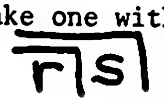
is equal to the unmarked state just as the initial says.

I proved it?

Sure, you said, the r has got to be either the marked state or the unmarked state. And then you tried both. And it came out to be the unmarked state in both cases. Which are the only two possible cases. That proof is good enough for me.

Yeh, I guess it's good enough for me.

Let's take one with two variables.



What we don't know is if it can
be simplified any further than this.

What can we do?

We can try all 4 possibilities.

What 4 possibilities?

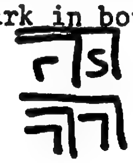
Well, r can be the marked or unmarked state
and so can s. So we have these possible
situations:

r is marked and s is marked
r is marked and s is unmarked
s is marked and r is unmarked
both are unmarked.

Oh, I see what you mean.
Do we have to try all four.

Yes. So we first try the
situation where both r and s are
~~un~~marked.

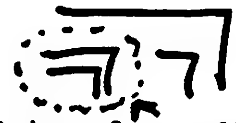
Right. We substitute the
mark in both places.



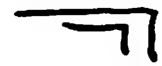
Then we start with the left-most
deepest place.

That's here

Apply the axiom of cancellation



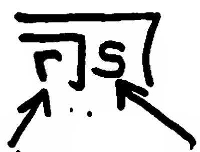
Axiom of cancellation
takes this out, leaving



Then apply the axiom
of cancellation again
and the answer is the
unmarked state.

Let's try it now for

r is marked and S is unmarked



Substitute the mark for r

...substitute the unmarked for s

That gives me

I'll apply the axiom of
cancellation to the
deepest left-most marks.

Hey, that leaves me with
the marked state for the
answer.

The last time we tried
it, when r and s were
both marked it came
out unmarked.

So you can conclude that

is the simplest that the expression can get when you
don't really know what r and s are.

Some of these just come out that way?

Yes.

What do you do about that

Nothing. Simply be content in the knowledge that you have the simplest expression you can possibly have. And that you've proved it.

Proved it! Did I do another proof that I didn't know I was doing.

Sure. You tried simplifying the expression and found, using the only possible alternatives, that it could not be made any simpler if the expressions were different and unknown.

Well, if you put it that way, I guess nobody could argue with you.

That's what a proof is.

Oh.

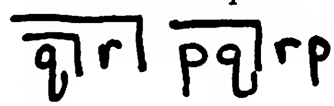
Now let's try some examples where you have to shuffle around some larger number of these variables. It'll show you how to use the consequences we introduced a while back.

O.K. But not too many.

Let's take anexample now and learn to apply the consequences to shuffle it around into the simplest possible form.

O.K., you got an example? Why should I ask? You always have an example. And it's always "just a little bith more complicated than the last one."

Here's an example:



I don't even know where to start.

Here's how I'd do it. I'd take a look at the table of consequences or the initials and see if there are any that seemed just like the situation you face here in all or part of the problem.

Where is the table of consequences and initials. What page did we leave them on?

Don't worry, we can reproduce them again here.

Thanks.

Ah, here's one. Look at the last part of the expression and then look at the consequence abbreviated gen.

$$\overline{\overline{q|}\,r|}\;\overline{pq|}\,r\,p \qquad \boxed{gen}\;\; \overline{ab|}\,b| = \overline{a|}\,b$$

Yes. That part looks similar. But what about that last p. can you just ignor that for the purpose of applying gen?

Yes. So what this means is that we can make

$$\overline{pq|}\,r$$

into

$$\overline{q|}\,r$$

by gen.

So our new expression look likethis:

$$\overline{\overline{q|}\,r|}\;\overline{q|}\,r\,p$$

question

Why does this work?

?

Well, you see that this really doesn't distinghish the p inside from the P outside. A distinction insxide and an distinction outside are like not having any distinction at all. That's the meaning of the consequence called ~~of~~ generation (gen).

So what do we do next?

We could try gen. again. Take this $\overline{q|}$ ~~and this r~~ which ~~are~~ is inside and ~~they are~~ it is equivalent to the $\overline{q|}$ ~~and r~~ on the outside. so we get

$\overline{r}|\ \overline{q}|\ \overline{r}\,p$

Then we ~~take~~ use gen again for this r and this r/and eleminate the r inside. And we end up with

$$\overline{\;\;}|\ \overline{q}|\ r\,p$$

Then notice that there is another consequence that is called integration and abbreviated int. It says

$$\overline{\;\;}|\,a = \overline{\;\;}|$$

And can be interpreted as saying, any mark outside a distinction just results in that distinction. It doesn't matter if a reduces to a mark or an unmarked state. The result wiill be a distinction.

That 's true of any time there is that form even if there is a whole bunch of things outside?

What if you had

$$\overline{\;\;}|\ \overline{\overline{a}|\ \overline{b}|\ c|\ de}$$

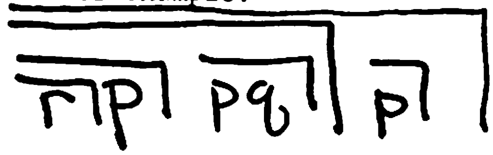
Could you eliminate all this stuff?

Yes, int permits that.

Wow! That makes things simple sometimes.

Yes. In this case what happens is that you can reduce it to the marked state .

Let's try another example.



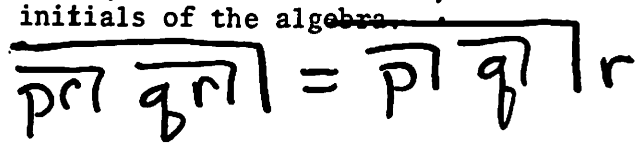
That's pretty big. I don't think I could handle it.

We'll try it step by step.

Let's try to do tansposition on this one first.

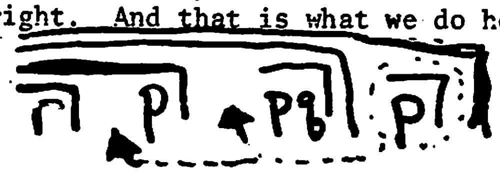
Which consequence is that?

It's not a consequence. IT's actually one of his initials of the algebra. this one:



What this says is that if you have something within a distinction twice, you can remove it. It is commmon to bothe of them. I doesn't really distinguish anything new.

You can go the othe way too remember. That is from left to right. And that is what we do here.

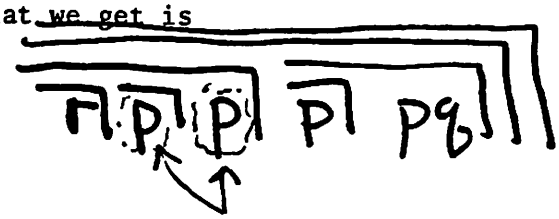


We take this p and put it in the two places indicated. So we are left with this expression.

You can do that with p inside a mark too.

Sure, its just an expression that will reduce to the marked state or the unmarked state at some point.

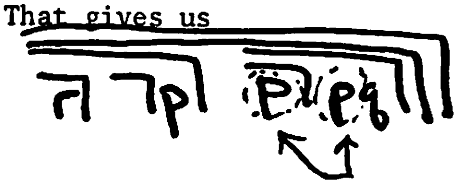
So, what we get is



Now let's use gen. here.

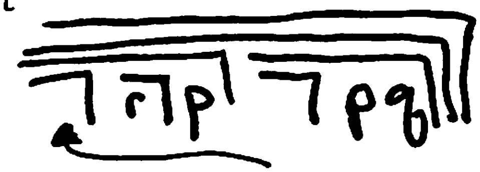
gen is ab⌉ b = a⌉ b

Right. That gives us



Then we can do gen. again on these two p's.

So we get



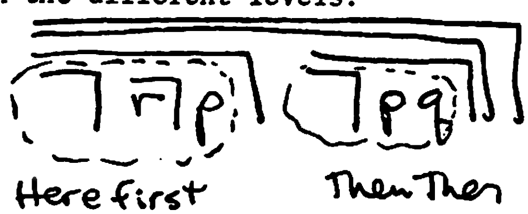
Note that he rearranged this mark

You can do that?

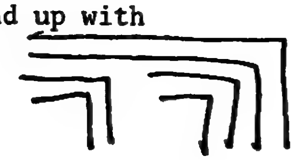
Sure, at the same level, or deapth, you can just move the order around.

Now we can use int. ⌉a⌉ = ⌉

within the different levels.

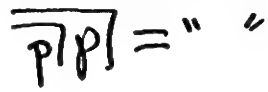


So we end up with



Then you can use the axiom of cancellation three times.

Or you could,doing the same thing, use position three times.



So you end up after all this with the unmarked state.

Well at least you get somewhere --or perhaps you could say you get nowhere, but you know where you got.

Yes, or something like that.

Chapter 5

Looking at Theorems

Theorems

First of all, let's review what a theorem is.

Yeh, review...if I ever really knew it in the first place.

Well,a theorem is a consequence.

It is an assertion that particular things are an inevitable result of the axioms you picked in the first place.

Let's see. You pick your axioms, such as the axioms of cancellation and condensation.

Then you deduce that certain things result from them.

That's right. You want to be able to prove that your theorems are always true.

All right. That all sounds like a very simple preamble to a difficult subject.

You'll find it isn't so difficult.

One of the things mathematicians do is to explore the properties or implications of a set of axioms. They want to be able to say "such and such" is true of all expressions (or a certain type of expression).

For example, you would want to be able to know if you can simplify any expression (consisting of a finite number of marks), before you started doing it.

Otherwise you might do a lot of work and end up without a simplification.

yeh

The answer to the question about whether you can simplify any expression consisting of a finite number of marks is "yes."

I proved that you can always simplify an expression if you have enough time.

But why prove theorems at all?

When you are proving theorems, you are exploring the properties of the expressions.

You are finding out about the constancy, invariance, condensation properties of the axioms.

What does that mean?

Some arrangements have a constant form. That is the arrangement doesn't go away no matter how you substitute variables in the expressions.

But still, why do the proofs?

Well, you want to know the constancies of arrangement before you start to apply a calculus. If you recognize the constant forms, and are absolutely sure of them, you make it easy for yourself.

Well, can you give an example.

Sure. Suppose you knew this about Spencer Brown's calculus. Supp ose you knew that with all possible arrangements there are only certain possible patterns that an expression could be reduced to.

Yes. So...

Well, you'd be able to do certain manipulations with the expressions with alot greater confidence. It's like having a map that someone has made of a place you're going. You're much more sure of the territory.

Stating a Theorem Formally

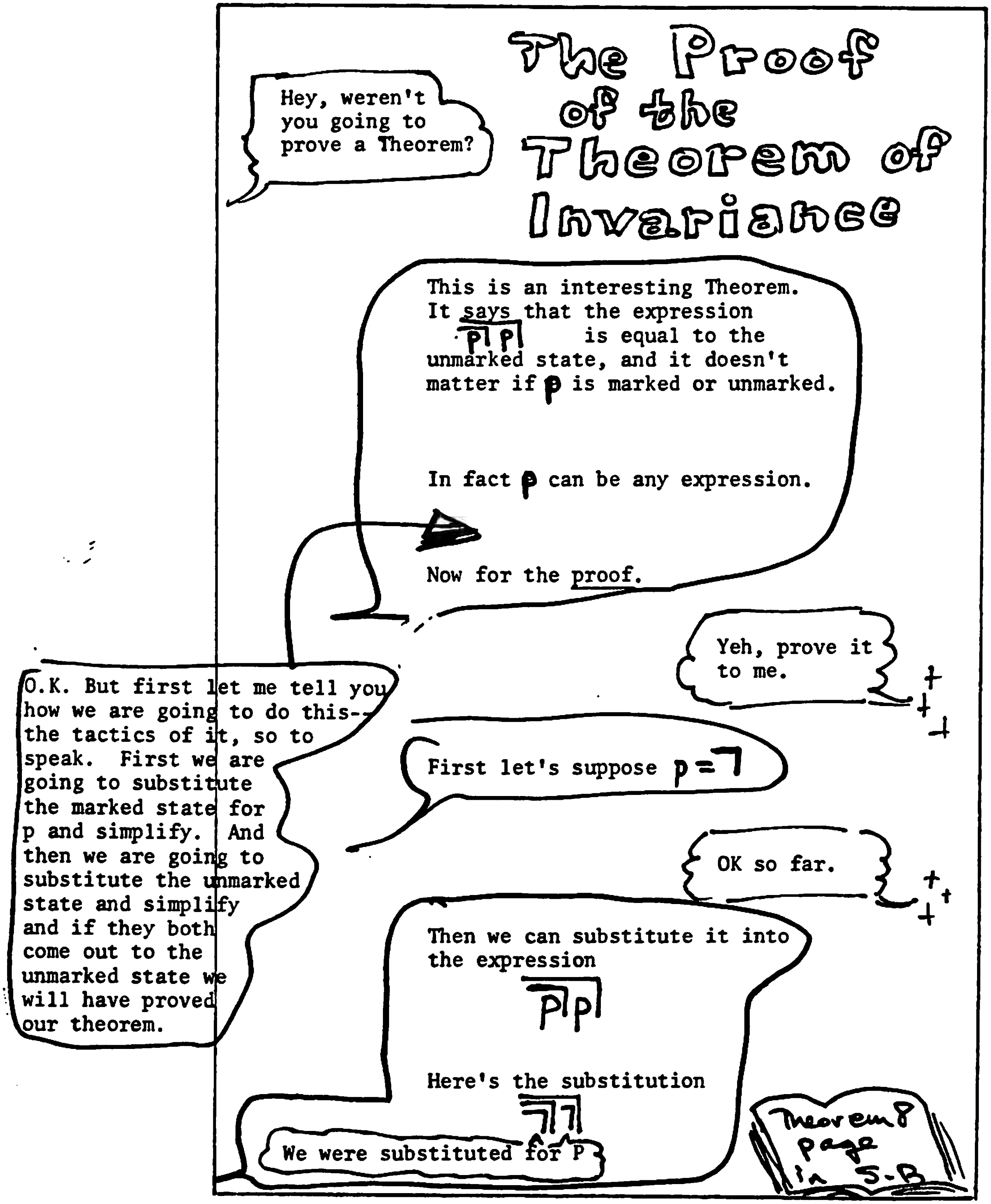
How do you state that last Theorem formally?

<u>Theorem 1.</u> An expression consisting of a finite number of marks can be simplified to a single mark or no mark.

We won't show you the proof for this one but it gives you the idea of what theorems are all about.

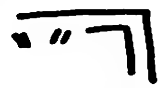
Will you prove some theorem?

Yes, but first we need some more notation.


The Proof of the Theorem of Invariance
Hey, weren't you going to prove a Theorem?
This is an interesting Theorem. It says that the expression ⌉p⌉p⌉ is equal to the unmarked state, and it doesn't matter if p is marked or unmarked.
In fact p can be any expression.
Now for the proof.
Yeh, prove it to me.
O.K. But first let me tell you how we are going to do this--the tactics of it, so to speak. First we are going to substitute the marked state for p and simplify. And then we are going to substitute the unmarked state and simplify and if they both come out to the unmarked state we will have proved our theorem.
First let's suppose p = ⌉
OK so far.
Then we can substitute it into the expression
⌉p⌉p⌉
Here's the substitution
⌉⌉⌉
We were substituted for P
Theorem 8 page in 5-B

We then apply the Axiom of Cancellation:

this cancels leaving



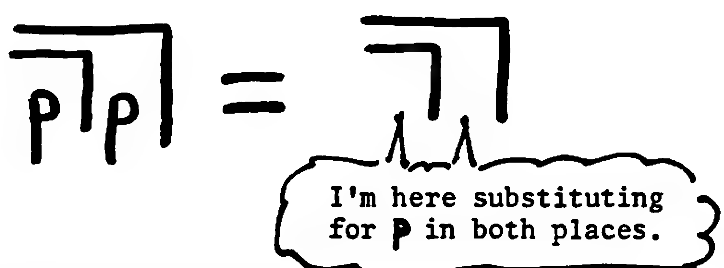
We drop the quote marks and we have

We apply the Axiom of Cancellation again and we end up with

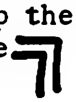
“ ”

Now let's try it the other way. Suppose we make p equal to the unmarked state.

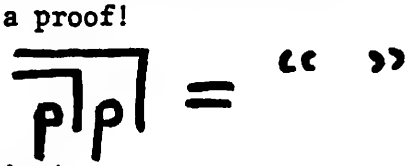
We substitute the unmarked state for p.



We drop the quote marks and we have



Then we cancel  and we are left with the unmarked state again this way. That's it -- a proof!



Everytime!

What other sorts of proofs are there?

There are some basically procedural proofs.

Theorems like:

Any expression can be simplified.

That seems kind of simple.

Yes, but you'd like to know it for any expression, no matter how big.

Yeh, I suppose you would.

And theorems like:

Any expression will come to the same result, no matter how you simplify it.

You'd want to know that in order to be sure that you wouldn't get two different results from doing the simplification of the expression in two different ways.

Yes. Any others?

Quite a few others. But here are a couple of useful ones.

Any even number of marked expressions in side of one another can be simplified to the unmarked state.

Any odd number of marked expressions inside of one another can be simplified to the marked state.

Is that true? If it is, it makes life alot simpler! All you have to do is count the funny little marks? And then you know how it is going to come out! Are you sure he proved it?

We'd better look at that. Note
that I said inside of one another.

Here's a demonstration.

We have three marks, and odd number

Let's apply the Axiom of Cancellation
to the inner two marks...

What do we end up with?

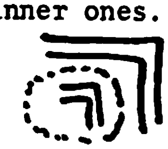
The marked state.

So, odd makes marked.

Now try an example of an even
number...

Four is even.

Apply the Axiom of Cancellation to
the two inner ones...



gives you

Drop the quotes and apply the Axiom
of Cancellation again...

And you get the unmarked
state. And even makes
unmarked.

I want to ask you a quesion.

Sure.

It's sort of a stupid question.

Anything you want to know.

Do you really need all this formalism, all these symbols? They make me uncomfortable.

Well, yes, I'm afraid you need them.

Why?

Well, they are recipes for creating thought. And thought creates many of our perceptions. That is, how we think determines how we see.

But what does that have to do with these unfamiliar symbols?

Well these symbols are recipes for getting you to think about areas where ordinary language is inadequate. We have no experience in some of these areas. And thus we have no language for them. That is true for much of mathematics and logic. They make clarity of thinking much easier. They're also very compact. You've seen how much longer and wordier the English is.

That's true. But I still sort of like the English.

That's one of the problems with using symbols. People want to relate to them immmediately to what they know. That is they want to treat mathematical and logical symbols as if they were English.

What's so bad about that?

If you do that, you get all screwed up.

Why do you get all screwed up?

Because in a mathematical system, the meaning is totally defined within the initial axioms. They are thought of as the defining axioms of the system you are dealing with. And nothing else.

If you try to attach other meanings to them, it is dangerous.

Dangerous?

Yes, because people try to attach all sorts of meanings to the notation.

You did that with the "Go through the door" example.

I did. And I felt I had to. But in a certain way I wish you'd forget the example and just think in the symbols.

But why?

Because the English--like any language--works because of its vagueness, because any word can have several shades of meaning. Language is the flexible communication tool it is, because of its ambiguity.

So...

Well, people want to treat notation like that too.

But they shouldn't.

No. They shouldn't. They should just take them to mean exactly as they are defined in the initial axioms.

So, what you're saying is that you get yourself an idea or purpose. You say with your axioms," here's what I want to think."

Then you prove a bunch of theorems. And you do that because you want to know what the consequences are of the thought system you made up in the first place.

Right.

And you think that behaving that way can be useful.

Sometimes it is.
Sometimes it's not.
Sometimes it's value is that the symbols are meaningless.

Why do you say that?

Well, because, sometimes there's a value in having to think something through in unfamiliar terms. You keep your thinking flexible. You enable yourself to think about new things.

Remember, I said, thinking often determines perceptions. So if you keep learning to think new ways, you may be able to see new things.

Actually <u>see</u> new things? Or are you just talking metaphorically.

Both.

Chapter 6

Introducing Self-Reference

Self-Reference

Logicians and ordinary people have long known that it is possible to create sentences that refer to themselves.

Example?

This sentence has exactly six words.

Another example?

This sentence has been written using the English language.

So what's the big deal?

Consider this example:

This sentence has exactly five words.

No it doesn't

Exactly. But consider the example:

This sentence is false.

Now, wait a minute. You can't do that?

Why not?

Well, if it is false then it is true. But if it is true then it is false.

Exactly the sort of problems that self-reference gets you into.

Well, it's a good thing we can avoid it.

Well, actually, it's alot harder to avoid than most people think.

self referential
There are alot of other/sentences I could show you. Alot of them are paradoxes.

O.K. Show me.

All rules have exceptions.

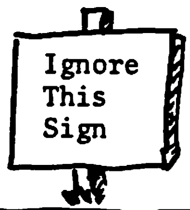
Including that one? If it does then it doesn't . If it doesn't,then it does . Oh, dear.

How about:

Never say never.

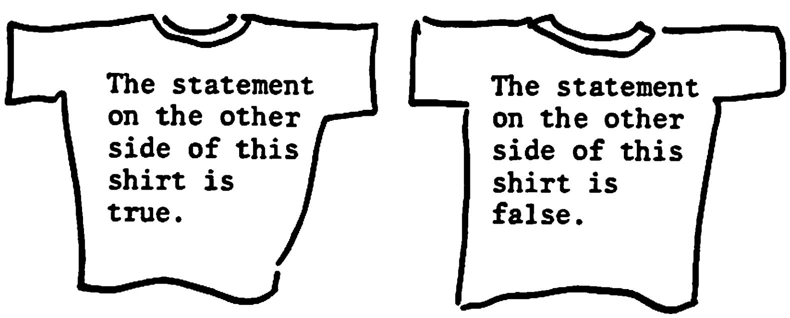
O.K. even I have used that one!

Here's a good one:



Hey, wait a minute. You have to read it, in order to understand it. But when you've understood it you've already violated the rule!

There are also two sentence self-referential paradoxes. Did you ever see this T-shirt?



Front Back

Lucky we don't have to deal with self-referential paradoxes in the real world.

Well, yes. But in alot of modern thinking in science and technology and mathematics dealing with them we have to deal with

- computer programs that use themselves to solve part of the problem

- functions that are functions of themselves

  We call these "recursive" functions or programs. It just means those that call upon themselves to do what they have to do.

There are also

- interactions that interact with themselves.

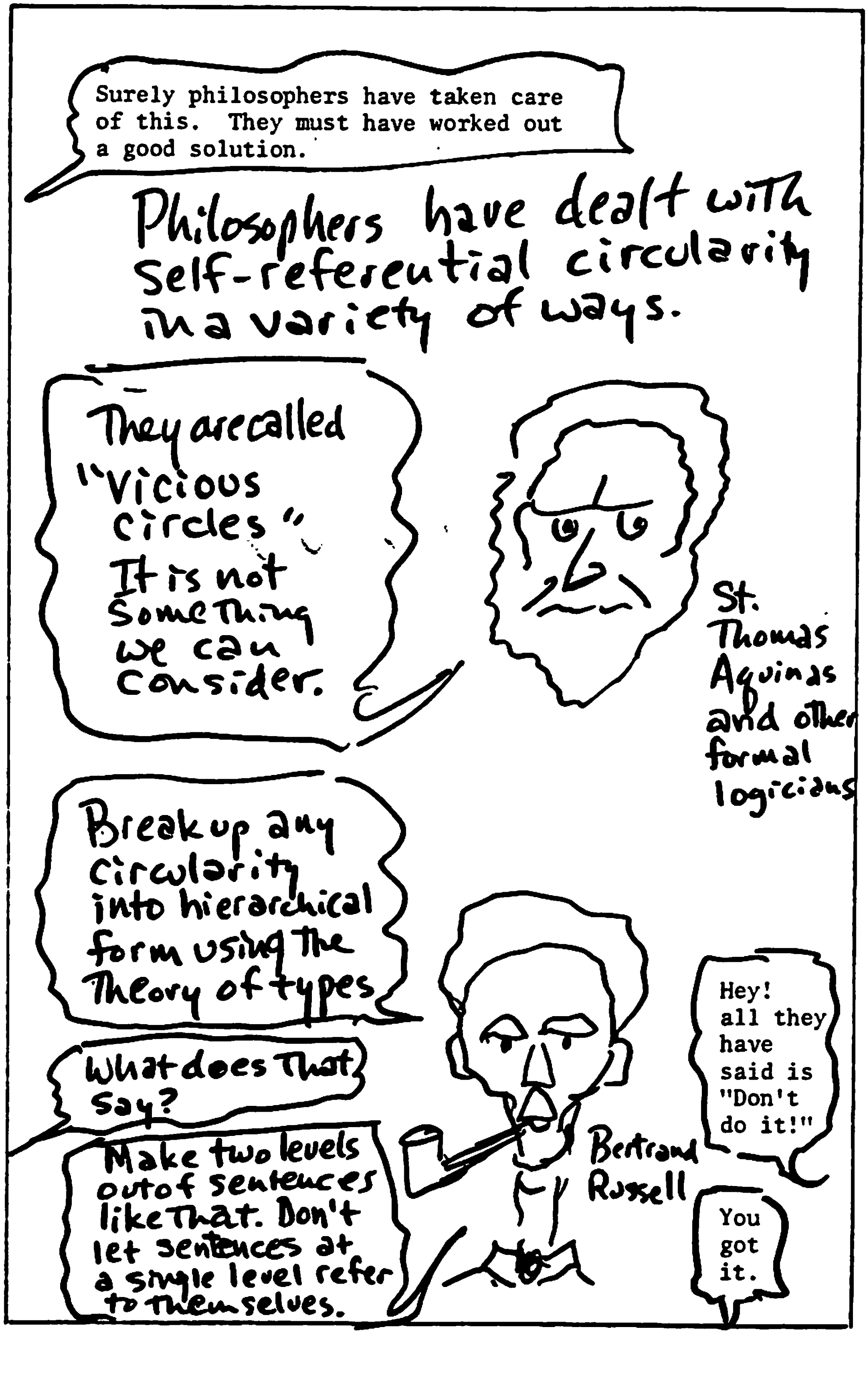
- systems that use themselves to produce other systems and to keep on producing their own existence.

Yikes! O.K. Maybe this self-referential stuff may be important. I thought I could ignore it.

- and biological systems

- and societies

O.K., O.K.! It could definitely be important.


Surely philosophers have taken care of this. They must have worked out a good solution.
Philosophers have dealt with self-referential circularity in a variety of ways.
They are called "vicious circles" It is not something we can consider.
St. Thomas Aquinas and other formal logicians
Break up any circularity into hierarchical form using the theory of types
What does that say?
Make two levels out of sentences like that. Don't let sentences at a single level refer to themselves.
Bertrand Russell
Hey! all they have said is "Don't do it!"
You got it.

Spencer Brown suggested that we consider what would happen in the domaine of distinctions (i.e. in the indicational calculus) if we introduce an expression that refers to itself.

> He brings it up after Russell and Whitehead said "Don't bring it up again?"

Yes

> Why?

Because he didn't see the necessity of introducing a new hierarchy of levels when it could be included in the calculus itself.

In other words, he has a way of reinterpreting the contradiction as a self-referential form...
He sees it as the completion or conclusion of all other logics.

> Can you say that it at least one more way, because I don't think I quite get what you're saying.

Instead of shutting down logic with two states, the mark and the unmarked, Spencer Brown allows a third state, which he calls the self-referent form or the reentrant form.

> You're going to explain all that, right?

Yes, in this chapter.
Let's start by introducing the expression that refers to itself...

Suppose we introduce an expression that refers to itself in Spencer-Brown's indicational calculus. It would look like this

$$i = \overline{i\,|}$$

Translate into English please.

OK. i is an expression that is equivalent to itself inside a mark.

So far, so good.

How are we to interpret this?

Does it have any meaning at all?

Let's see how it behaves.

If we say that $\overline{i\,|} = i$, it means that we can substitute $\overline{i\,|}$ for i.

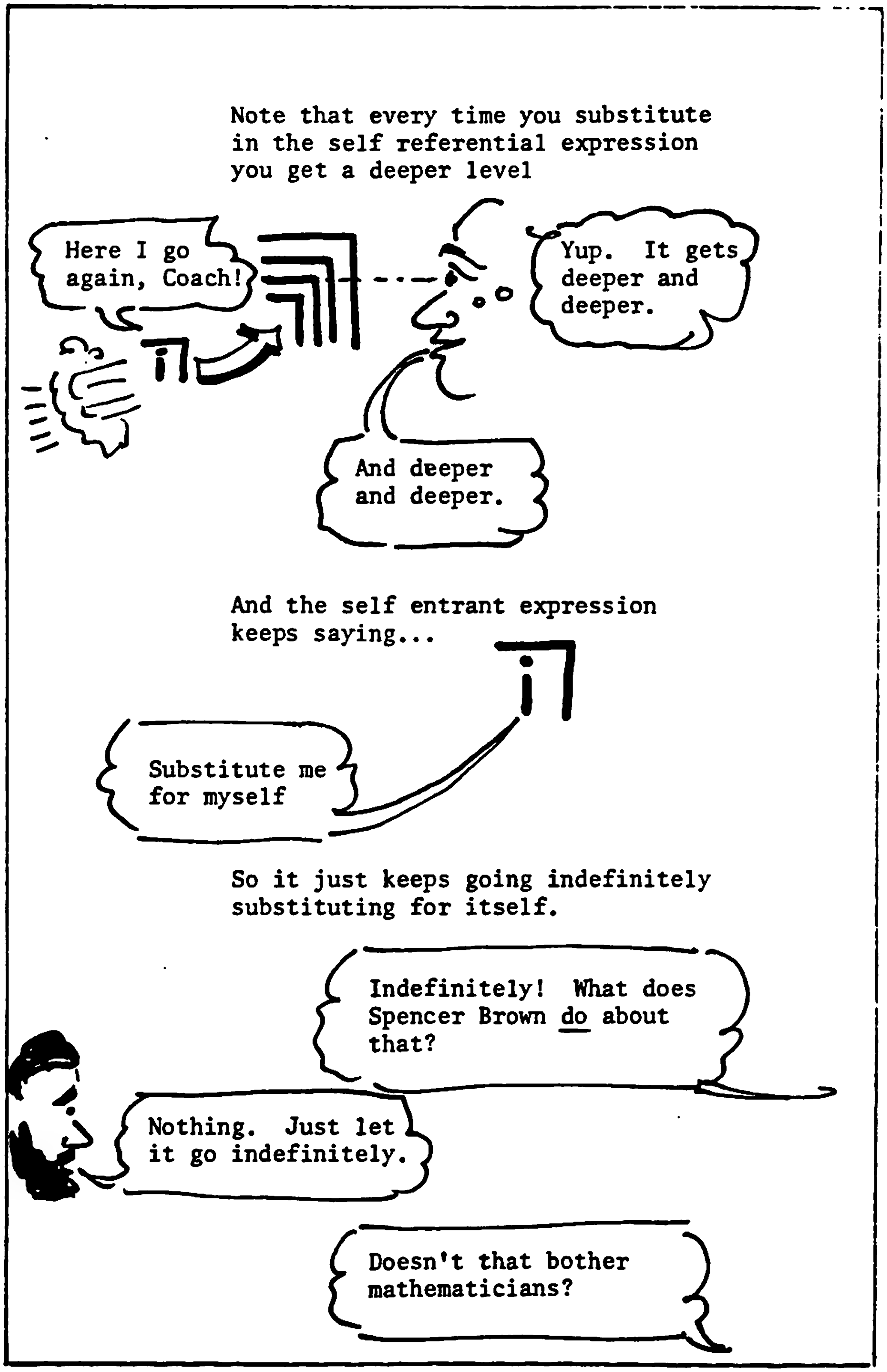
So let's do it.

$$\overline{\overline{i\,|}\,|}$$

OK

But we can keep doing that because $i = \overline{i\,|}$, we can keep putting $\overline{i\,|}$ in the expression.

Hey we could keep doing that forever!


Note that every time you substitute in the self referential expression you get a deeper level
Here I go again, Coach!
Yup. It gets deeper and deeper.
And deeper and deeper.
And the self entrant expression keeps saying...
Substitute me for myself
So it just keeps going indefinitely substituting for itself.
Indefinitely! What does Spencer Brown do about that?
Nothing. Just let it go indefinitely.
Doesn't that bother mathematicians?

Yes we could keep doing it forever.
Let's introduce the symbol three dots
(...) for this could keep going forever.

and ever and ever...

So we can write

$$i = \overline{\overline{\overline{i\,|}\,|}\,|} \ \ldots$$

What is the value
of this expression?

We don't know its value.
So, because it is indefinite,
i has become <u>indefinite</u>
or indeterminate . Why? Because its
value depends on how many marks there are in it.
And to know that we have to know how many marks.
But we don't know. So we say it is in determinate.
We can just keep going DEEPER and DEEPER and NOT STOP!

Oh dear--
what do we
do about that?

Right now we aren't going to
do anything about that. Just
notice it. We just have to recognize
that it is irreducible to either the marked state
or the unmarked state. It has no determinable
value...

But, we can look at the self referent
expression in another way.

You always have
another way of
looking at it!

Note that i can have two possibilities the marked state or the unmarked state.

Suppose we start with the self-referent expression $i = \overline{i\,}|$.

Then we apply the Axiom of Cancellation ($\overline{\overline{\;}|}| =$ " ") to it. We get

i and $\overline{\;\;}|$ can be cancelled

to " " so

$i =$ " "

What! You can't have that--that's contradictory. It says "a mark is not a mark!"

That's right. Or it says

Mark - not mark - But mark - not mark

$i =$ " " $= i =$ " " $= i =$ " " ...

Three dots for sure! Indefinite again! Jeeze! This is silly. What possible good can you make of this mess?

There is yet another way to look at the self referent expression. Francisco Varela suggests that all descriptions are based on distinctions. Hence, they are at bottom 2 sided (this and not that).

This would mean that all of the categories we think in are at bottom similar to the mark-not marked states. He points out that

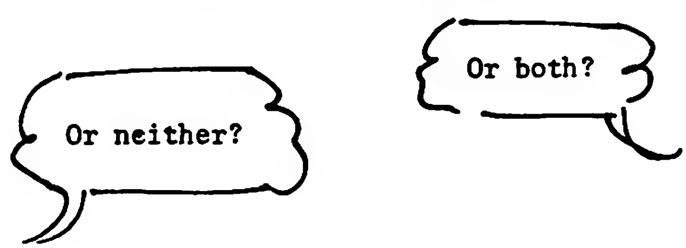
pattern/dynamic

is one such basic complementarity of thought.

A pattern is static. It exists in space.

A dynamic is a moving pattern. It exists in time.

So we can ask, what does the self-referent form distinguish? Is it a mark for a pattern or for a dynamic.



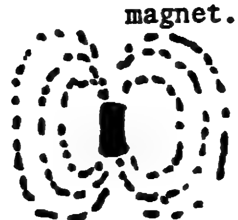
Varela points out that the patterns or dynamics that come from the self referent expression behave a lot like the way we think of

patterns in space	dynamics in time

First let's take it's representation in space.

This version of the self referent mark yields a coherent pattern something like a group of

forms in nature that repeat themselves indefinitely, such as lives of force around a magnet.



Or, the electromagnetic waves sent out from an oscillator

Or the waves in a pond...

Another representation of the self referent mark can be made in time,

i = " " = i = " " = i ...

which could be symbolized by an ocilatory wave (by substituting ⊓ for ¬| and ___ for " ")

It would look like this

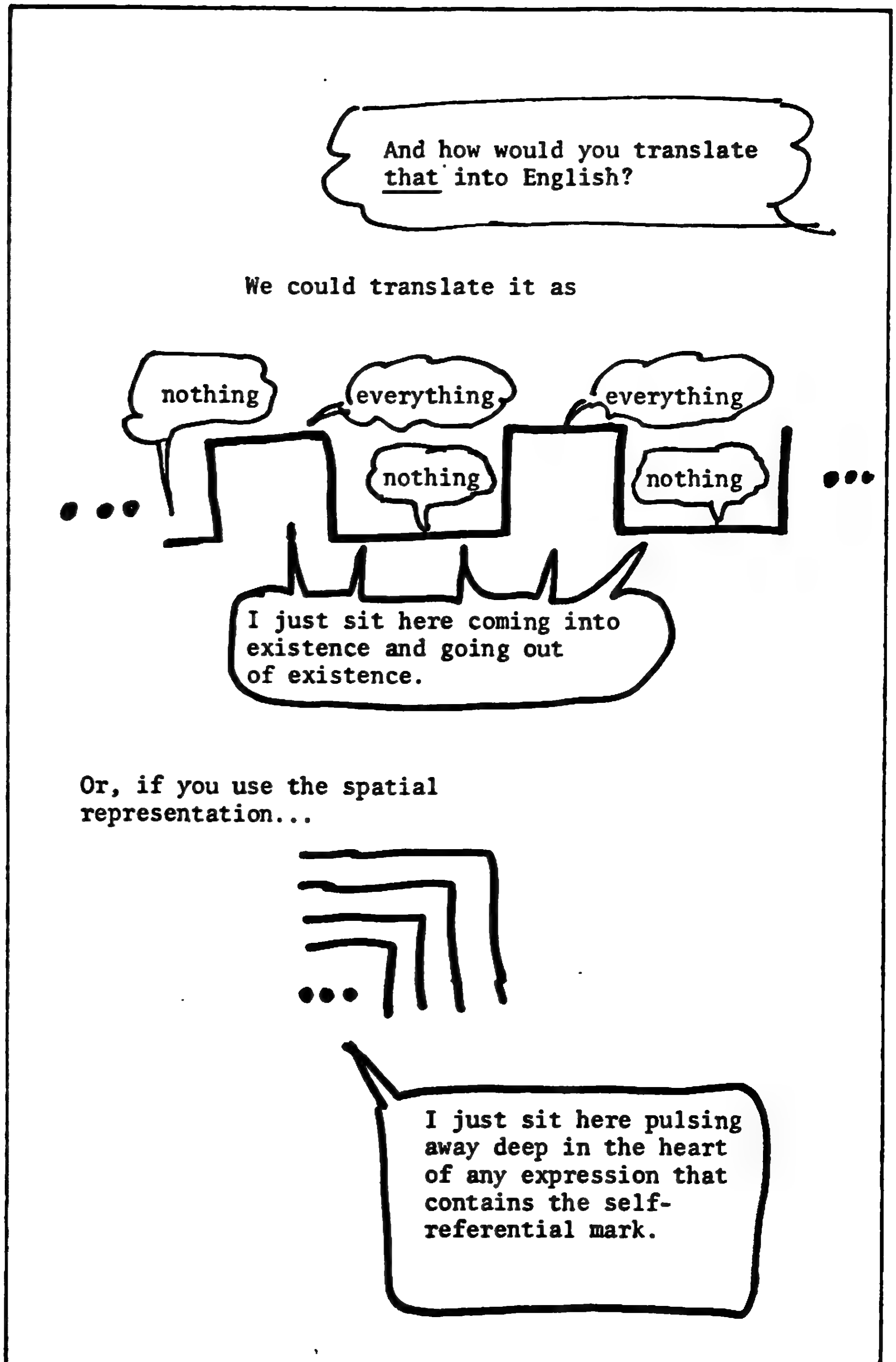
It's like
On-off-on-off-on...

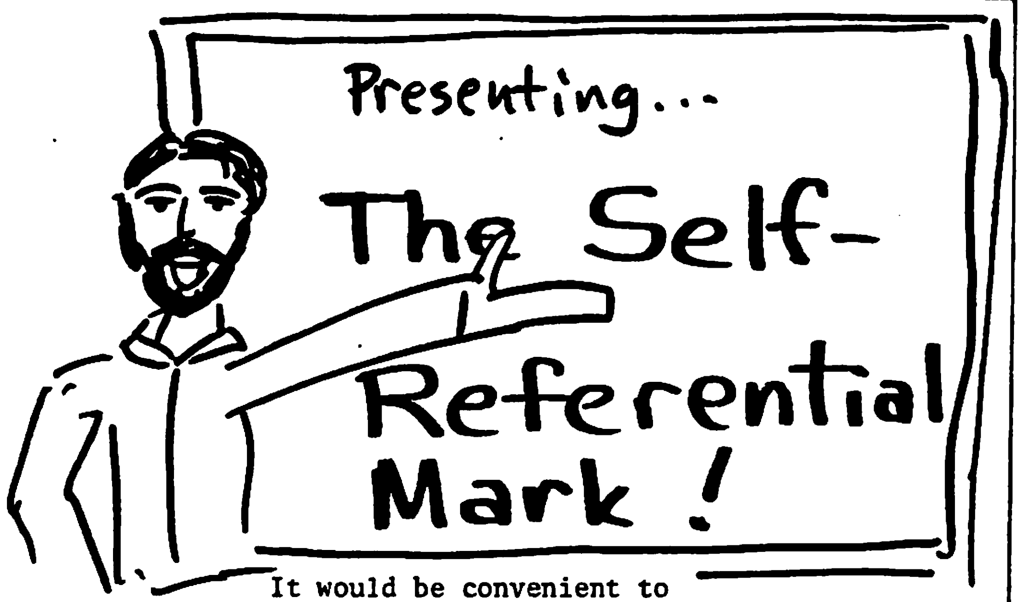
Here we take the reentrant expression

i = i¬|

and let i = ¬|

in which case using the Axiom of Cancellation you get
i =" "
and then substitute that result
i = " " for
i in the expression i = i¬| like this i = " "¬|
which reduces to the marked state. So in a serie of timed substitutions, we get


And how would you translate that into English?
We could translate it as
nothing
everything
everything
nothing
nothing
I just sit here coming into existence and going out of existence.
Or, if you use the spatial representation...
I just sit here pulsing away deep in the heart of any expression that contains the self-referential mark.



It would be convenient to have a way of avoiding certain repetitive and cumbersome expressions that come about from the use of self reference in the indicational calculus.

It sure would!

So Spencer-Brown has introduced the self-referential mark.

He calls it the mark that "reenters" itself.

Or the reentrant form
or the reentrant distinction.

So, when you are considering a self-referent expression, you are considering the 3d value an expression can have.

Before we had the self-refernt expression all you could do is reduce the expressions to a marked or unmarked state. Now there is a 3d value.

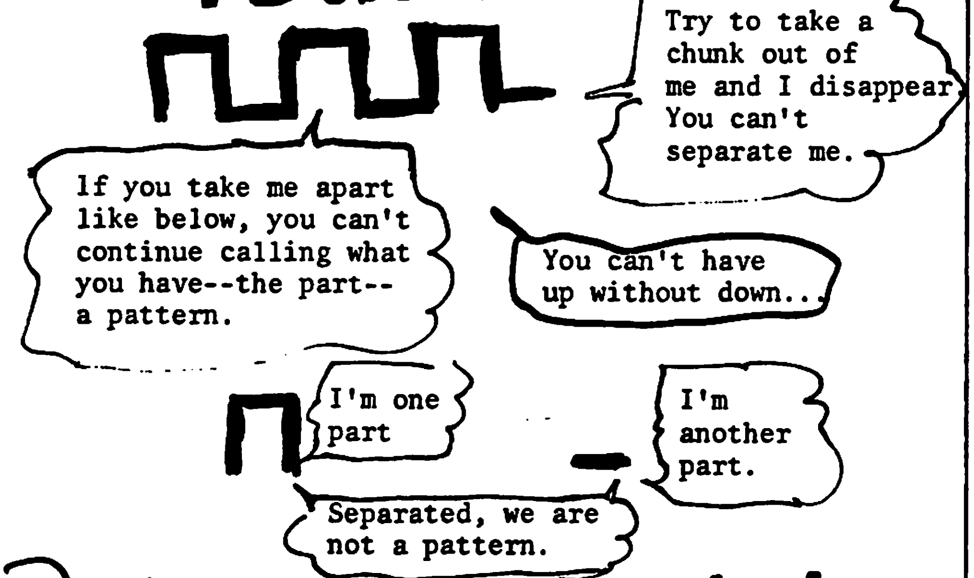
And, it unfolds into an infinitely repeating pattern in space and time.

...a holograph in space...

...a wave form in time.

And it is a UNITY. That is

You **can't separate** a **Pattern**



Patterns are **wholes**

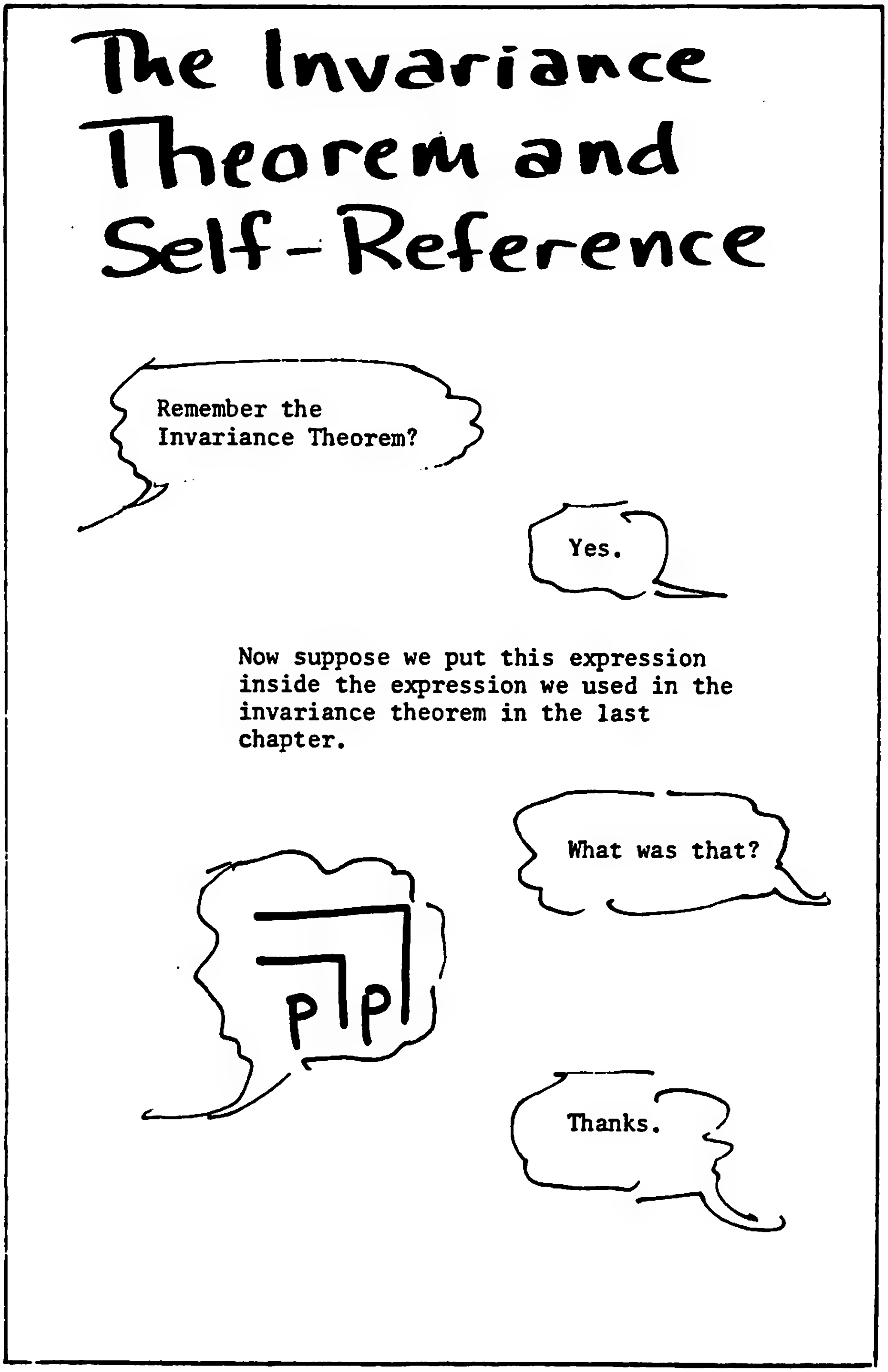
One more thing. We note how

a unity arrises from self-reference.

Note:
One of the greatest philosophers and mathematicians of the 20th century, Alfred North Whitehead, regarded each element of the universe as

...a vibratory ebb and flow of an underlying energy or activity... ultimate elements are in their essence vibratory

Alfred North Whitehead


The Invariance Theorem and Self-Reference
Remember the Invariance Theorem?
Yes.
Now suppose we put this expression inside the expression we used in the invariance theorem in the last chapter.
What was that?
p p
Thanks.

How do you put this $i = \overline{i\,|}$
inside the $\overline{\overline{p\,|}\;p\,|}$?

By substitution. We substitute
i for p
and we get

$$\overline{\overline{i\,|}\;i\,|}$$

OK I see the
substitution.
But, so what?

Let's simplify.

$$\overline{\overline{i\,|}\;i\,|}$$

We can apply the Axiom of
Cancellation $\overline{\overline{\;\,|}\,|} =$ " "
and get $i\,i =$ " "

Then we substitute $\overline{i\,|}$ for i
twice and get

$$\overline{i\,|}\;\overline{i\,|} = \text{" "}$$

Apply Axiom of Condensation $\overline{\;\,|}\;\overline{\;\,|} = \overline{\;\,|}$
and get $\overline{i\,|} =$ " "

Hey wait a minute! I thought $\overline{i\,|} = i$!

That's right. So we get a
paradoxical result when we
put a self-referential
expression in a theorem of
invariance.

Like this sentence is untrue?

Yes.

So what does Spencer Brown
do with that problem?

He just lets it be.
He says that down at the bottom of
making distinctions that refer
to themselves is an indefinite,
infinite pattern or marks and
no marks.

Well, at least he doesn't
say, Don't do it.
the way Russell did.

He just leave it there...
flip-flopping along...
back and forth...
vibrating away...
Hmmm...

Chapter 7

The Biology of Self-Reference

Did you know your brain contains about 10^{10} cells?

That's alot. But I keep forgetting just how to do the scientific notation. What does ten to the tenth mean?

It means a very large number. It means 1 followed by ten zeros. Altogether that's 10,000,000,000 or ten billion cells.

Whew! That's alot to think about.

And each cell contains about 10^5 macromolecules.

What's a macromolecule?

Well, it's a very large molecule, made up of alot of smaller ones. So, your brain contains about 10^{15} macromolecules.

Hmmm. That's 1,000,000,000, 000,000! That's even more to think about.

By the way, that figure may be off by aboutone order of magnitude.

Remind me again, what's an order of magnitude?

Well, in the case of macromolecules, its the difference of 100,000,000,000,000. That is, you take off one zero for each order of magnitude.

So my brain may be 100,000,000,000,000 molecules smaller than I just thought it was

Yes.

!!! But, why all this talk about macromolecules?

Well, it has been estimated that your brain gets rid of and obtains new macromolecules for each cell an average of 10^4 times during a lifetime. That means that according to the biologist Paul Weiss "every cell in your brain actually harbours and has to deal with 10^9 macromolecules during its life."

That's alot of turnover. But so what?

Did you ever notice that, despite the turn over, you still remember things.

Oh, yes. I guess that is remarkable.

Weiss says "despite that ceaseless change in dtail in that vast population of elements, our basic patterns of behaviour, our memories, our sense of integral existence as an individual, have retained throughout, their unitary continuity of pattern."

O.K. I get it. How do we do that?

Nobody quite knows the answer to that question. But some biologists are quite interested in the question.

The fact that we retain our autonomous unity of body and mind despite the turnover of the components of our cells is referred to as

autopoiesis

which can be translated as "self-producing," or "self-creative," or "self-renewing." The word "auto" in Greek means "self" and the word "poicsis" means "creating." (The adjective form is "autopoietic.")

Humberto Maturana and Francisco Varela, two Chilean biologists and system theorists, have used that word to refer to any system that has self-creating properties.

It's their word for "living" systems, right?

I guess you'd say that. And what's interesting, especially to Varella is that autopoietic systems somehow do what they do by self-reference.

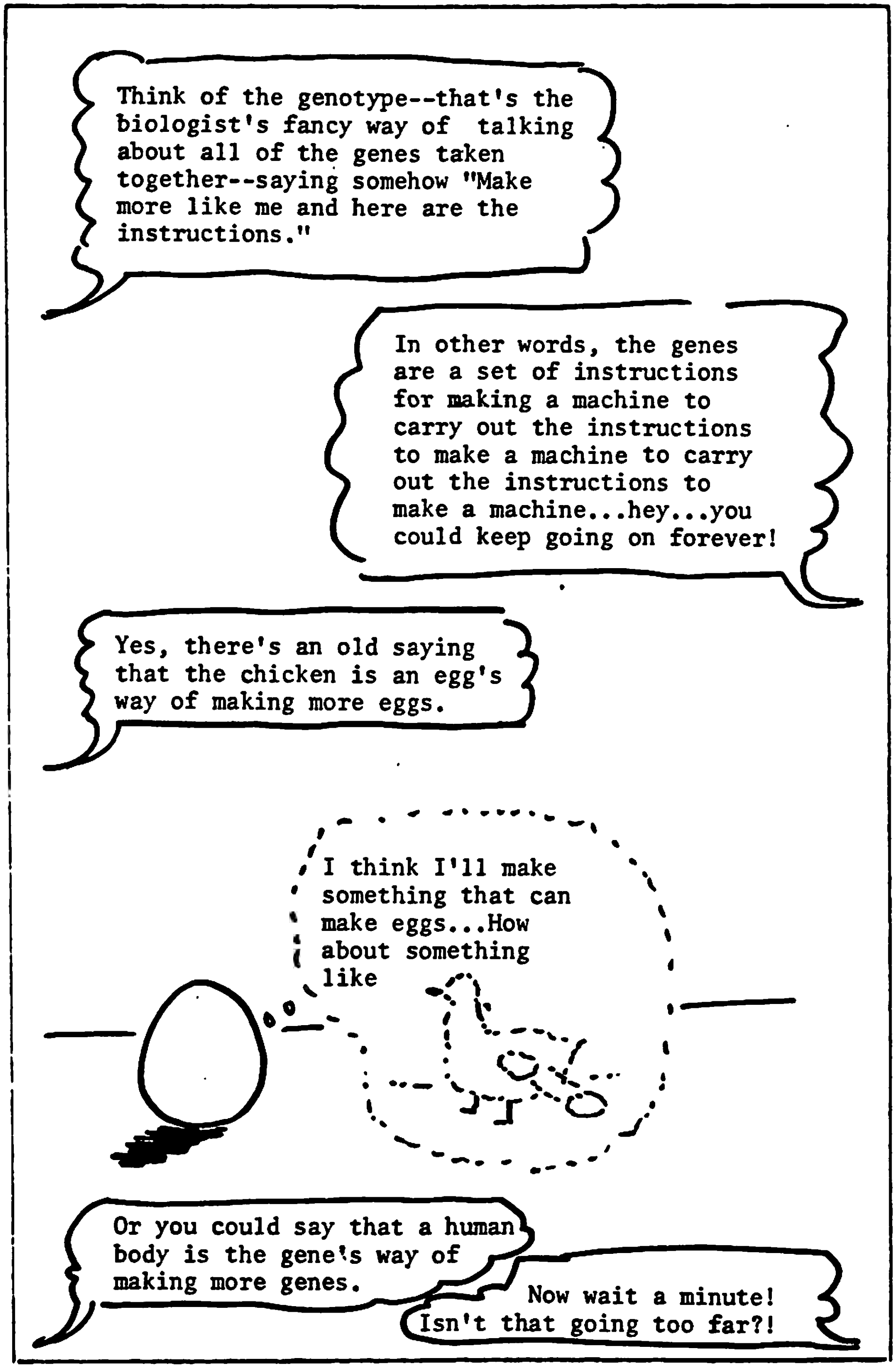
What do you mean?

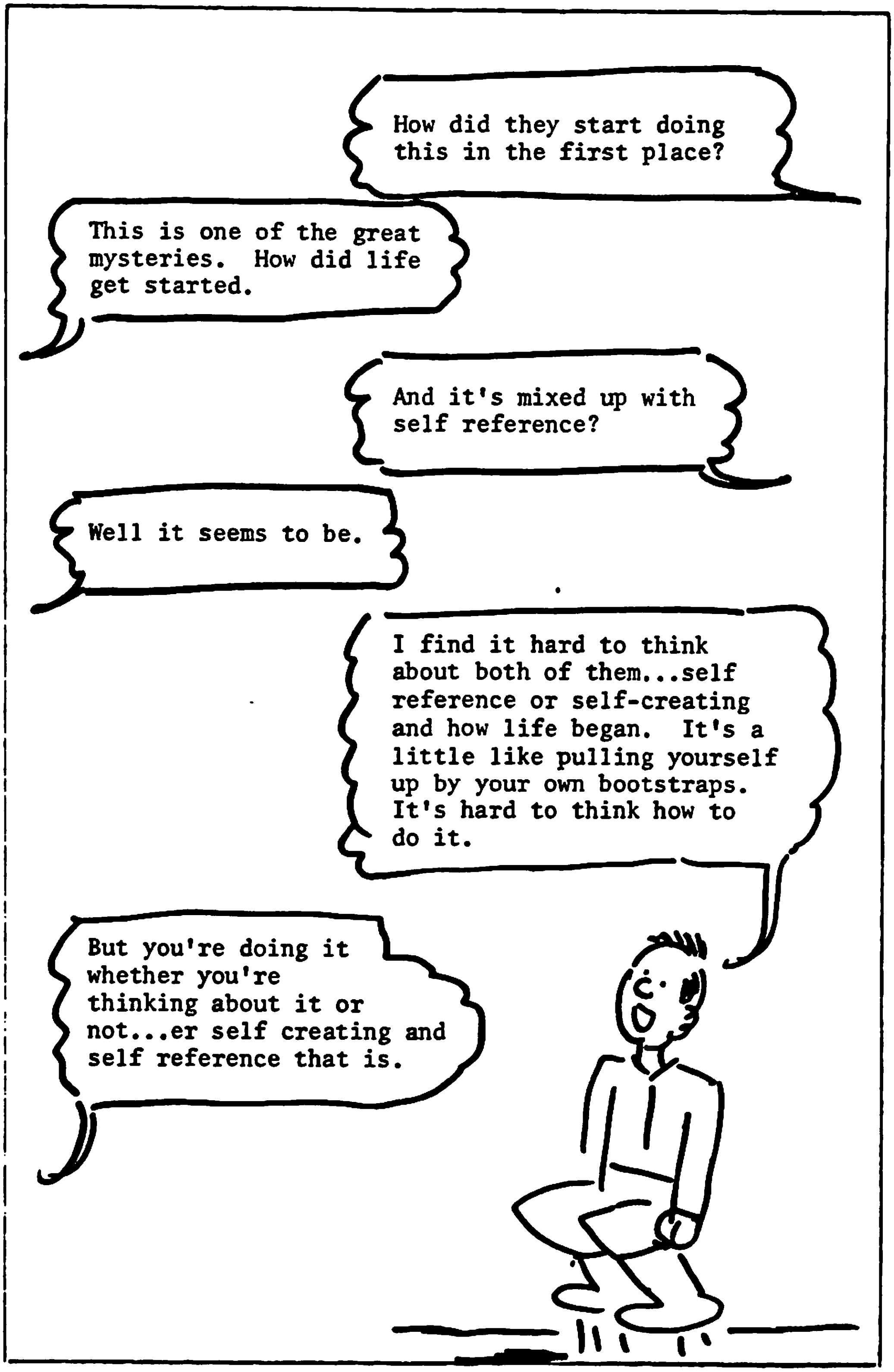
Well, somehow, they are continuously maintaining their integrity, their boundaries; and they are continuously creating more of themselves both by reproduction and by continuously keeping themselves going.

Why is this self referential?

Somehow, they are giving themselves the message: "keep making an organism like the one who is saying this. And I am saying this to myself."

That's what we say to ourselves! That sounds a little mixed up--but all those self-reference messages sound a little strange when you write them in English!


Think of the genotype--that's the biologist's fancy way of talking about all of the genes taken together--saying somehow "Make more like me and here are the instructions."
In other words, the genes are a set of instructions for making a machine to carry out the instructions to make a machine to carry out the instructions to make a machine...hey...you could keep going on forever!
Yes, there's an old saying that the chicken is an egg's way of making more eggs.
I think I'll make something that can make eggs...How about something like
Or you could say that a human body is the gene's way of making more genes.
Now wait a minute! Isn't that going too far?!


How did they start doing this in the first place?
This is one of the great mysteries. How did life get started.
And it's mixed up with self reference?
Well it seems to be.
I find it hard to think about both of them...self reference or self-creating and how life began. It's a little like pulling yourself up by your own bootstraps. It's hard to think how to do it.
But you're doing it whether you're thinking about it or not...er self creating and self reference that is.

How do you go about thinking about self-reference? How would you use it in looking at problems in biology?

Well, first you'd look at where in the process is there self-reference.

But, you'd find that everywhere!

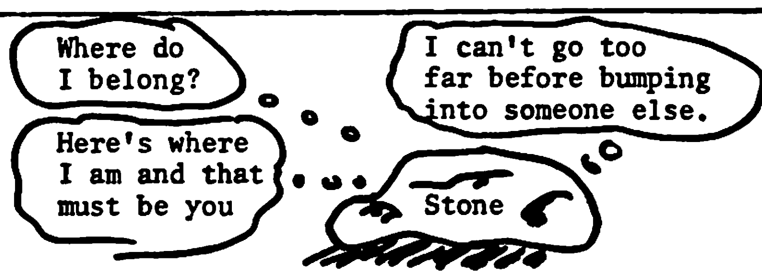
That's right. Any process that maintains itself with feedback contains self-reference.

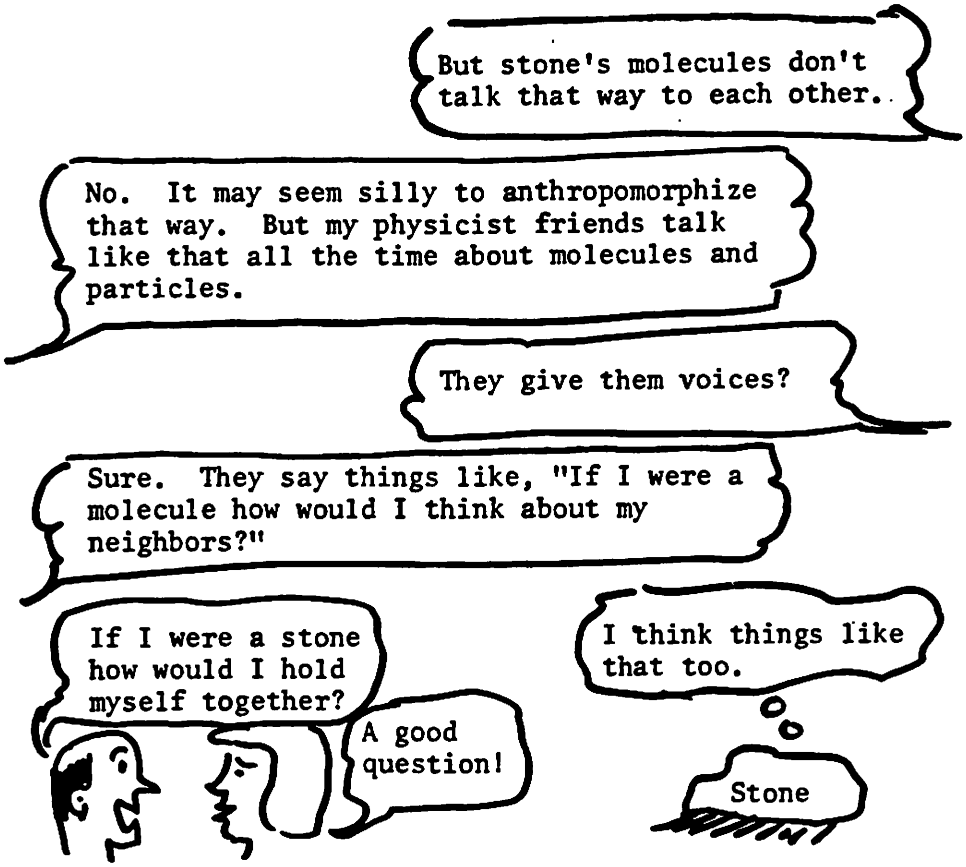
How does it do that?

Well, that's often what you are trying to describe in biology. Somehow it has to give itself feedback as to whether it is itself (otherwise how does it maintain its integrity, its boundaries?)

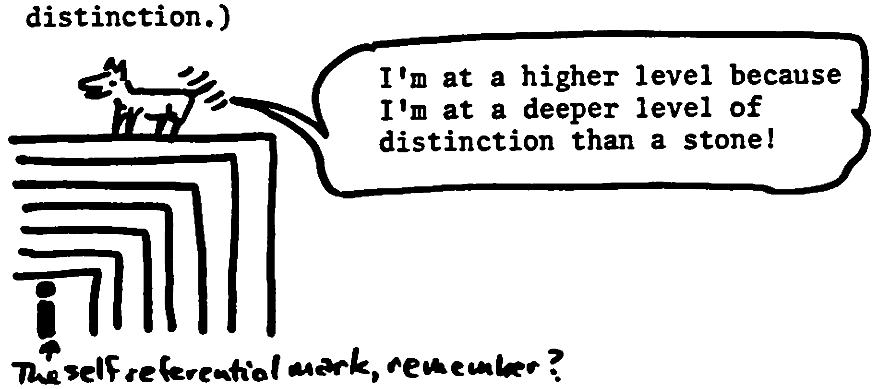
I can see that in animals and plants, but what about inanimate objects?

Maybe even a stone has feedback. The molecules inside a stone are coupled to each other. They have some kind of primitive feedback to each other at their boundaries. It's a low level of feedback. It's internal and invisible. But somehow the stone's molecules are saying things to each other.





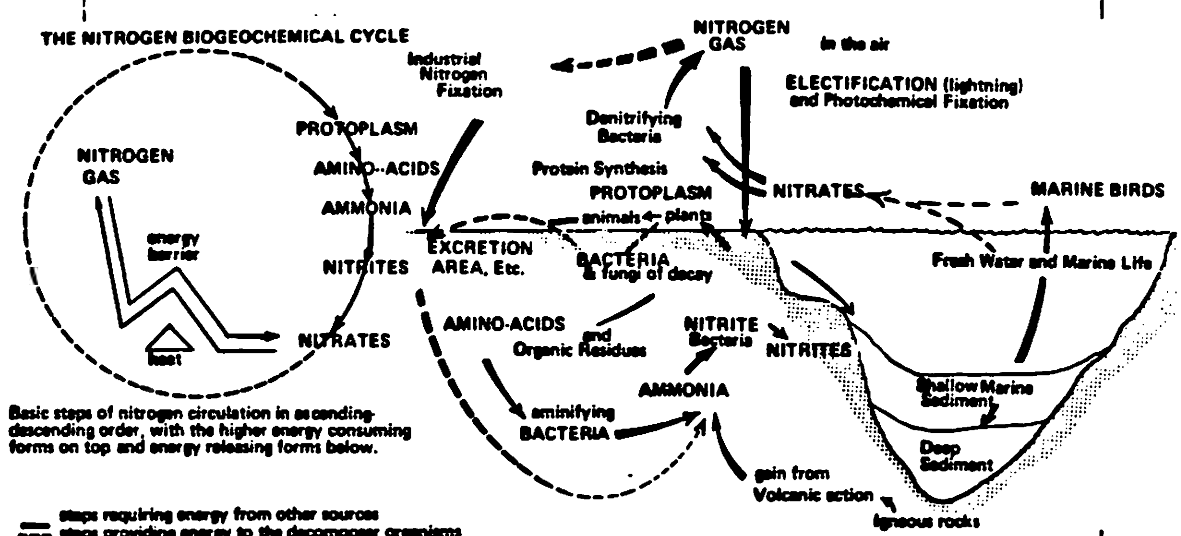
Larger, more complex organisms have larger feedback systems that enable them to maintain more complex structure. This means that we can say that the self-reference is at a higher level in the order of distinction. (If you think in terms of the depth of penetration of the distinction.)



In biology, we note that there are self referential loops all over the place.

I don't usually see them.

The eye is not very good tool for observing them. The eye sees only what is here right now. The eye can not see many of the patterns in nature. It does not see the vast cycles such as the nitrogen cycle:



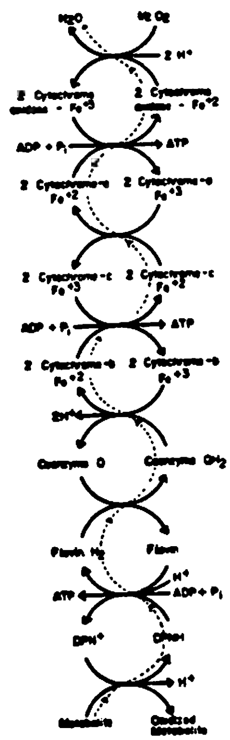
You can't see this. It is a visual model of ideas based on observations of discrete events. But the whole cycle has self referential loops as characteristics.

I've seen such charts in ecology books. How does it have self referential characteristics?

Somehow, it maintains itself. It is on-going. It keeps itself going and is self correcting in certain respects. It maintains the boundaries of the system.

There's another process that is very self referential. Chemists call it the metabolic pathways.

Figure 4.
The Electron Transport System



From ESSENTIALS OF BIOLOGICAL CHEMISTRY by J. L. Fairley and G. L. Kilgour.

Note that in these processes many times the product of one process feeds into itself. And there is the characteristic that it keeps on going--not just any way--but in a way that has an identity, a predictible one that chemists can describe and predict.

And all that from messages to itself to behave in some predictible way.

Messages is an anthropocentric metaphor. But, yes, something like messages perhaps.

You're hedging.

Sure. We don't know just how to talk about these things.

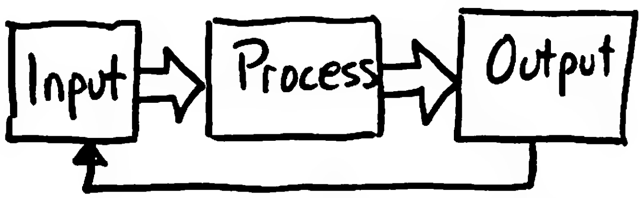
But people still write books about them.

Yes.

Another way of talking about self-reference is to notice that biological organisms are wholes--and to describe the nature of wholes adequately you have to deal with self-referential descriptions...systems that create themselves.

One of the ways we have described biological processes is to use the input-process-output model.

We assume that a biological process looks like this

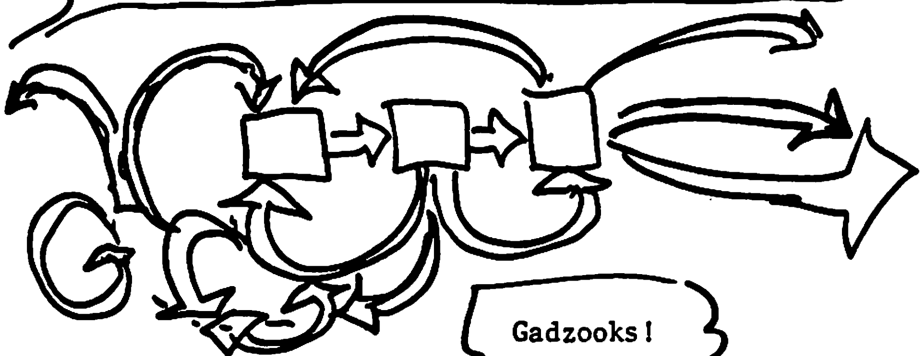


That's classical systems theory.

That's right. That's how most scientists and cyberneticists think.

But you can't do that for a living thing.

First of all they would be better represented like this (with all their loops)



Gadzooks!

Besides that there is no beginning. You might say the beginning is the end.

But certainly the input-output diagram is useful for something.

Of course! It was the basis for gigantic stries in understanding phenomenon and it is still tremendously useful in designing systems in an engineering sense as well as studying natural self-referential systems so long as you can deal with vast simplifications in the self referential properties.

This actually takes us up against the problem of how did life first begin.

That's a big question!

Well, some biologists work on it. Of course, one of the questions is how did the first cells know how to become cells?

Yes, how?

We don't know.

Oh...

But, somehow the cell had to get itself together out of what the biologists call the molecular soup (which is just alot of molecules mixed together like some alphabet soup).

But the real trick is that the cell had to specify its boundaries.

How did it do that?

Presumably through certain molecular productions that produce a boundary.

And how did "the molecular productions" come about?

They come about when there are cell boundaries.

Hey, wait a minute. Did you just say something like "the boundaries permit the creation of molecules that have the capacity of creating boundaries?"

Yes.

But which came first, the boundaries or the molecules that permit the creation of boundaries

that permit the creation of molecules

Yes, it's like"which came first the chicken or the egg?" Which came first, the molecules that permit the creation of boundaries or the cell boundaries that permit the creation of molecules that permit the creation of boundaries...

So do biologists have an answer?

No. But once such a situation occurs, a whole new domain comes into being--life. And life, it appears, is quite involved with such self-referential loops. Once you're into self-reference,you're talking about the producer producing the producer producing the producer...

And alot of the distinctions we usually make in thinking about things don't make too much sense...

Such as?

Such as"beginning"and"end"

yes, which came first...

and"producer"and"product"

O,K. I see. whatever you are talking about is both producer and produced.

and"input"and"output"

I guess you can't make a clear distinction between them. You have to say the cell wall and the molecules are both input and output

So, how do we talk about this subject, if everything we say is all blurred together , all tangled into knots.

All "deeply intertwingled," as Theodore Nelson likes to say.

Intertwingled! Scientists don't use that word,do they?

No, but I like it.

But you asked how do we talk about this new deeply intertwingled knot of self-reference. We are just learning how to talk about it. G. Spencer Brown's calculus of indications is one such probe... one such exploration. It helps us keep things straight at a very simple level. It may not be the actual language that we use to work on many of these biological problems. Or it may be that someone will come up with a way of using it or extending it to deal with them.

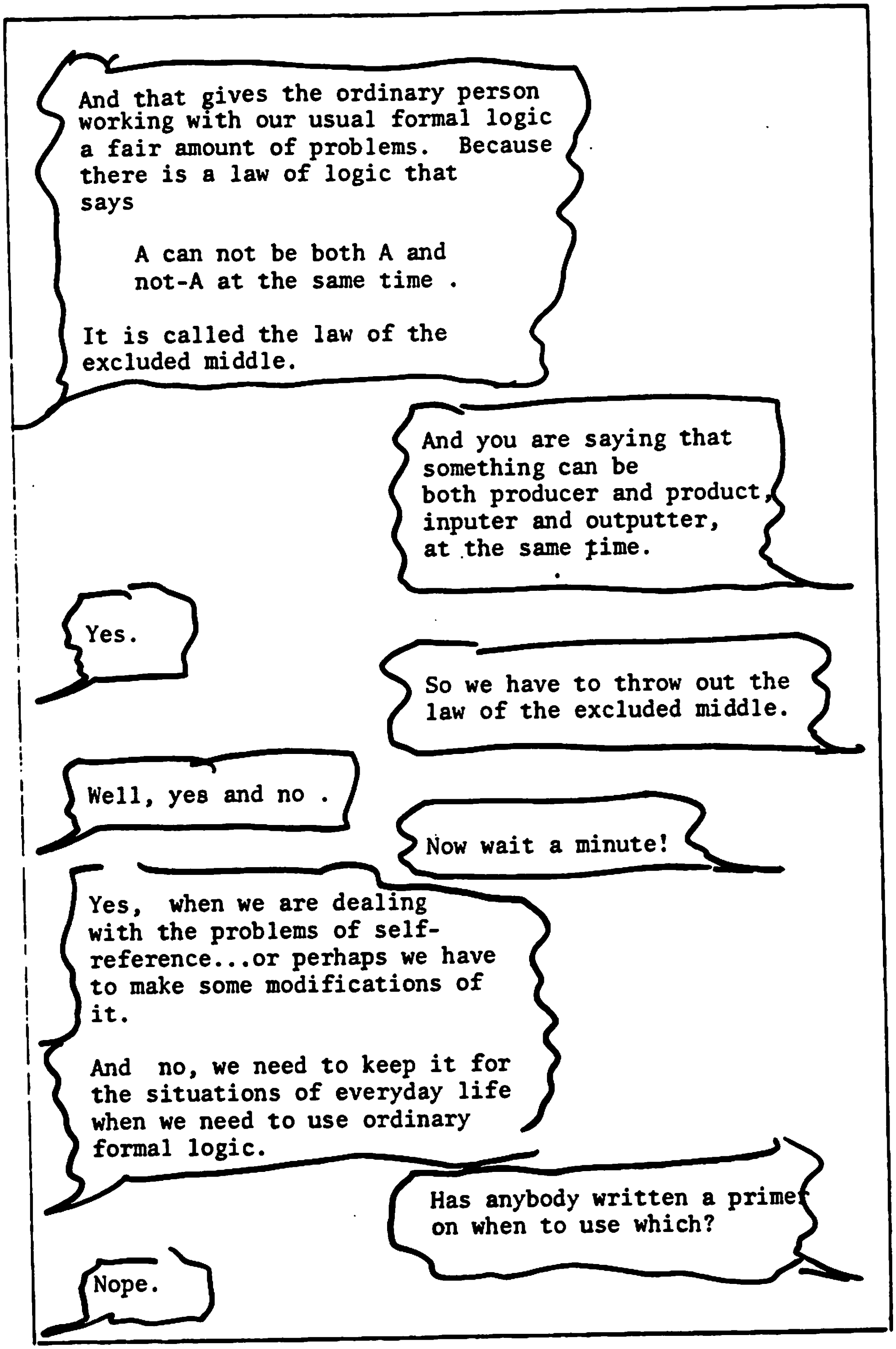
Very modest. I'll buy it.

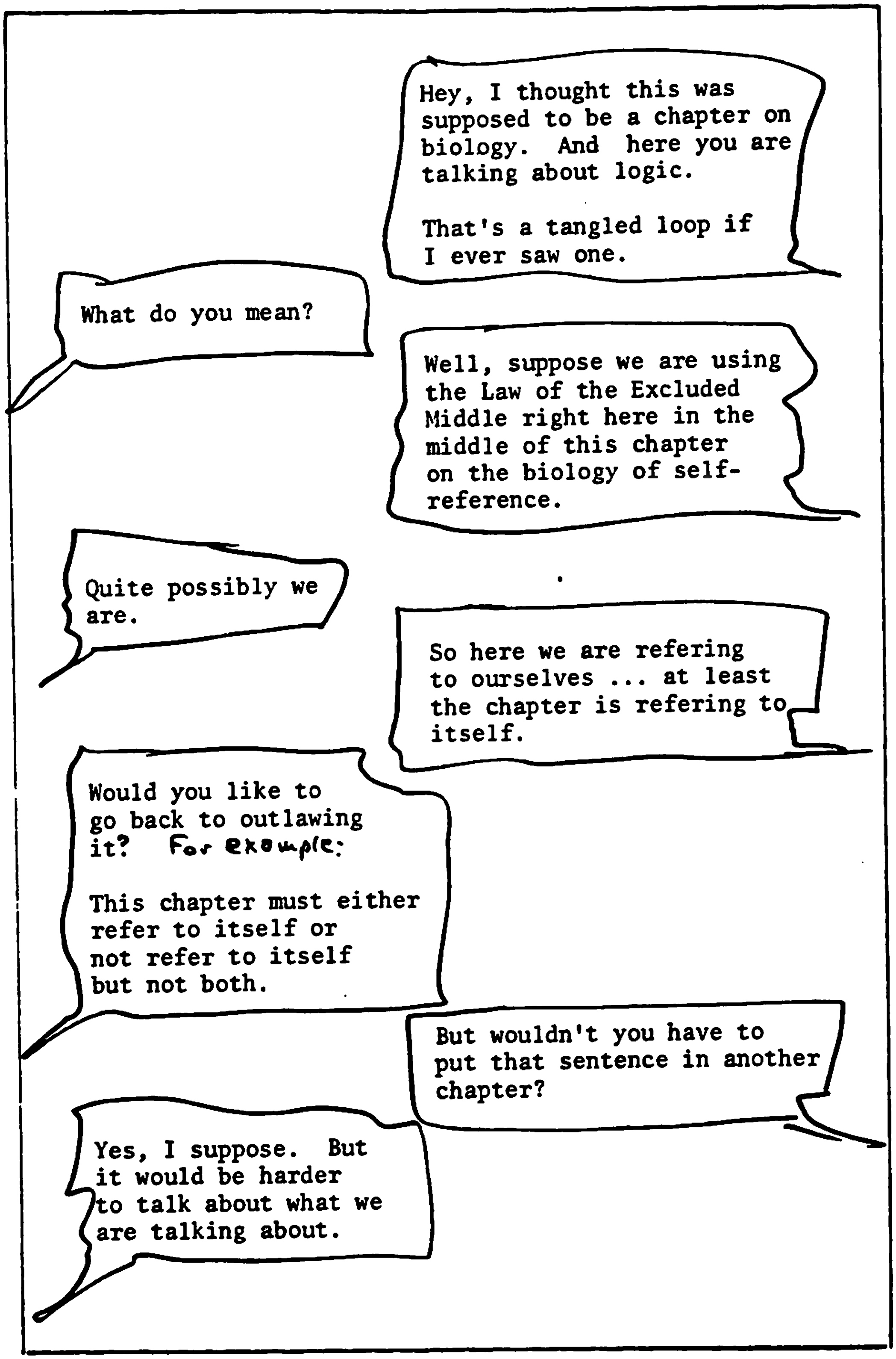
But what we do find that in these kind of situations we have to watch out for our usual logic.

Such as?

Such as when we say "the cell and the boundary can't both come first. It must be either one or the other."

Yeh, I see what you mean. In this case"both-and"sure beats"either-or."


And that gives the ordinary person working with our usual formal logic a fair amount of problems. Because there is a law of logic that says
A can not be both A and not-A at the same time .
It is called the law of the excluded middle.
And you are saying that something can be both producer and product, inputer and outputter, at the same time.
Yes.
So we have to throw out the law of the excluded middle.
Well, yes and no .
Now wait a minute!
Yes, when we are dealing with the problems of self-reference...or perhaps we have to make some modifications of it.
And no, we need to keep it for the situations of everyday life when we need to use ordinary formal logic.
Has anybody written a primer on when to use which?
Nope.

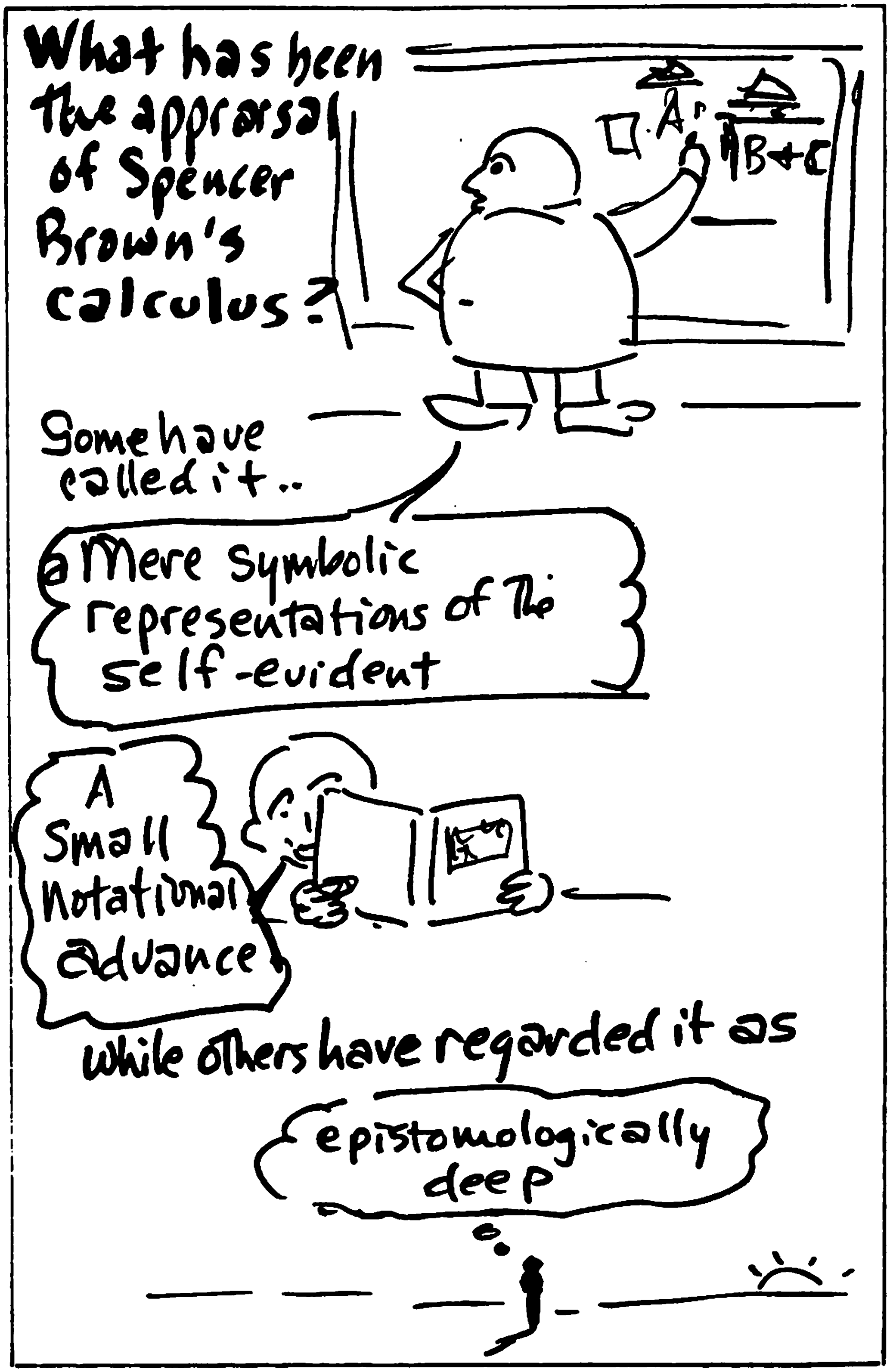

Hey, I thought this was supposed to be a chapter on biology. And here you are talking about logic.
That's a tangled loop if I ever saw one.
What do you mean?
Well, suppose we are using the Law of the Excluded Middle right here in the middle of this chapter on the biology of self-reference.
Quite possibly we are.
So here we are refering to ourselves ... at least the chapter is refering to itself.
Would you like to go back to outlawing it? For example:
This chapter must either refer to itself or not refer to itself but not both.
But wouldn't you have to put that sentence in another chapter?
Yes, I suppose. But it would be harder to talk about what we are talking about.

Chapter 10

The wider significance

larger smaller narrower

deeper? shallower inner outer


What has been the apprarsal of Spencer Brown's calculus?
Some have called it..
a Mere symbolic representations of The self-evident
A Small notational advance
while others have regarded it as
epistomologically deep

The importance of Spencer Brown's calculus is that it is

1) the foundation of systems theory

2) the foundation of mathematics

3) a simple model for studying self-reference — which is difficult to study in natural systems

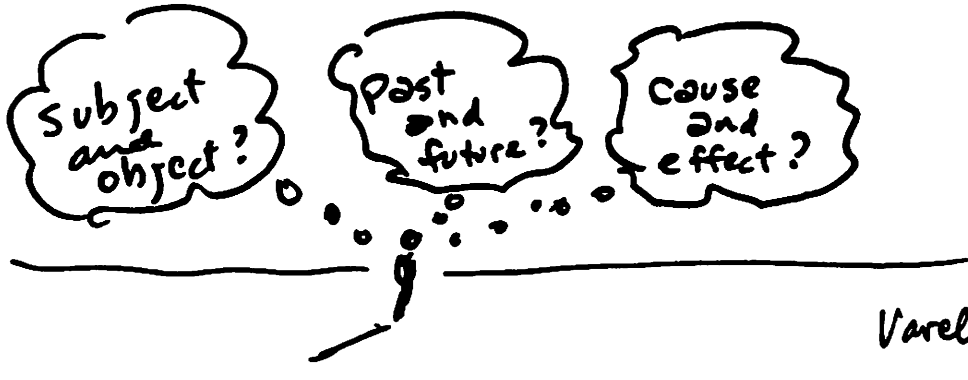
Francisco Varela

Varela goes on to point out That The indicational calculus is A non-dualistic attempt to set foundations for mathematics and descriptions

Spencer-Brown's vision, then, amounts to a subversion of the traditional understanding on the basis of descriptions. It views descriptions as based on a primitive *act* (rather than a logical value or form), and it views this act as being the most simple yet inevitable one that can be performed. Thus it is a nondualistic attempt to set foundations for mathematics and descriptions in general, in the sense that subject and object are interlocked.

Varela

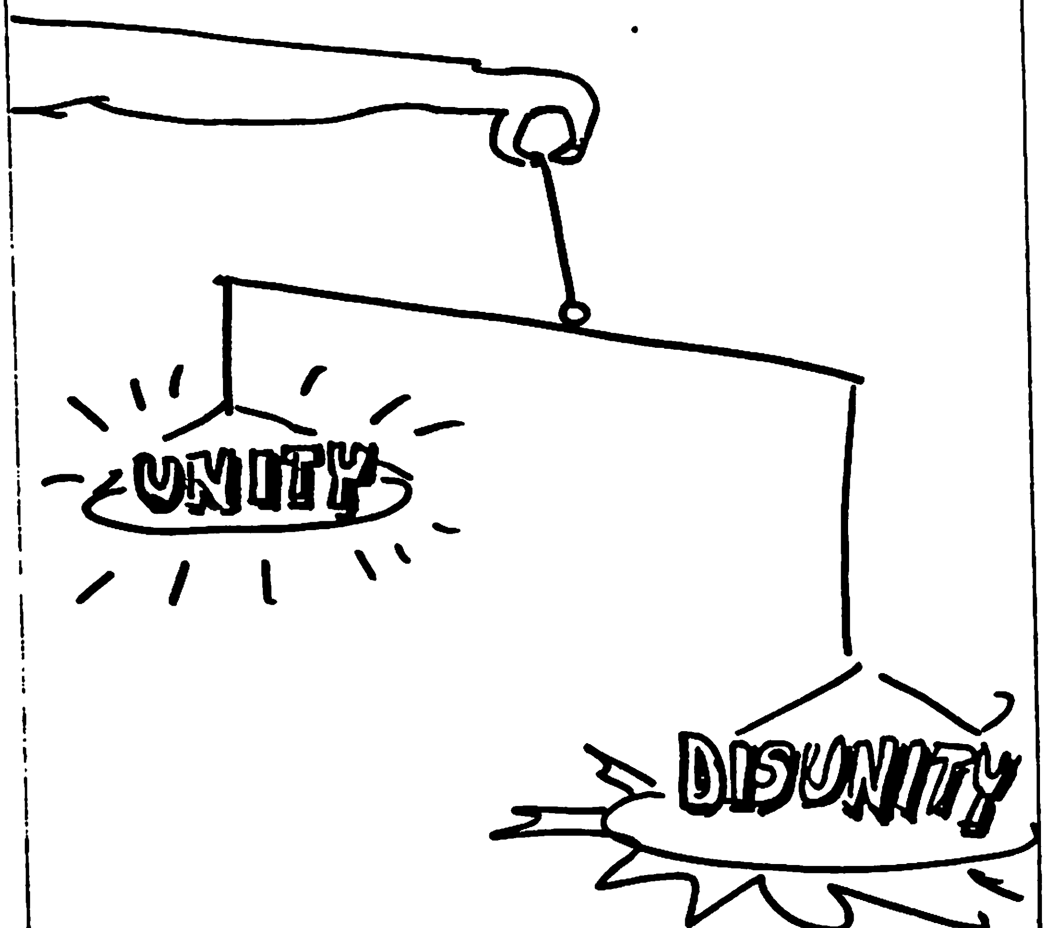
i.e. because the mark can be both an operator and operand, interchangably, it is non-dualistic. It avoids The maker/made describer/described dichotomy at its most basic operation.



Varela

In another sphere...

On the scale of planetary concerns... many people feel That our **unity/differentiation** Complementarity is badly out of balance



Humanity has had the talent to

Manifest Splits

in a most prolific fashion...

Manifesting splits, differences, disunity seems to be much easier to do than to create unities

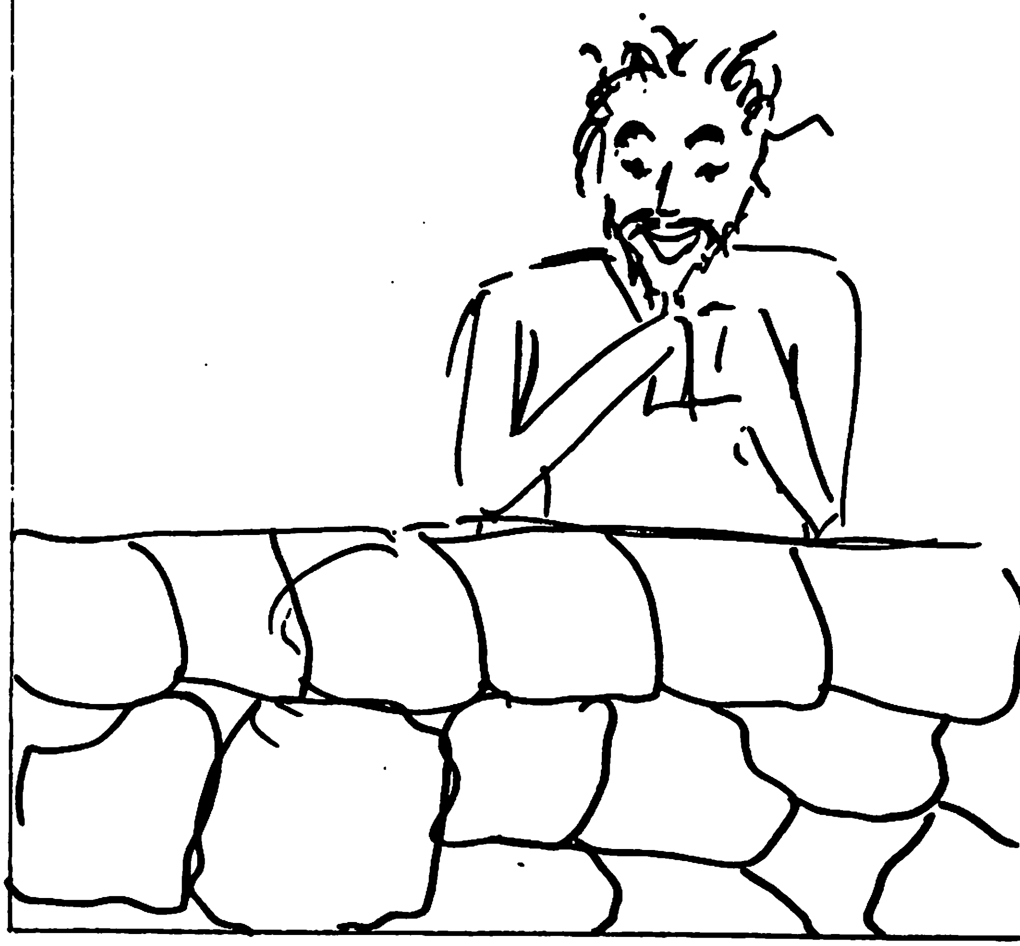
LAO TZU, the ancient chinese sage:

Is there a difference between yes and no?
Is there a difference between good and evil?
Must I fear what others fear? What nonsense!
Having and not having arise together
Difficult and easy complement each other
Long and short contrast each other
High and low rest upon each other
Front and back follow one another.

author of The Tao Te Ching

And Chuang Tzu, another important Chinese philosopher...

Thus, those who say that they would have right without its correlate, wrong; or good government without its correlate, misrule, do not apprehend the great principles of the universe, nor the nature of all creation. One might as well talk of the existence of Heaven without that of Earth, or of the negative principle without the positive, which is clearly impossible. Yet people keep on discussing it without stop; such people must be either fools or knaves.



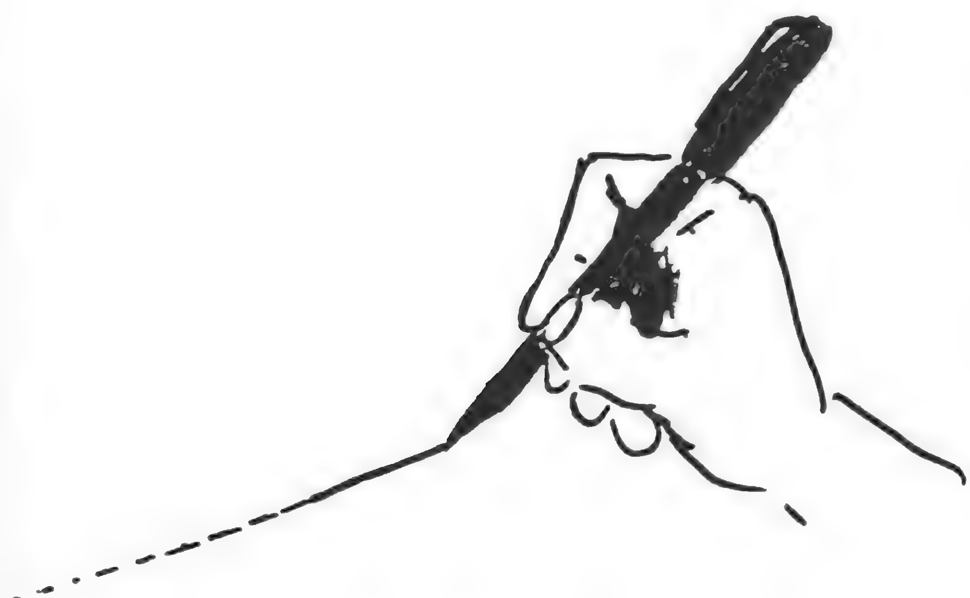
And the contemporary philosopher Paul Williams

We are learning not to draw lines.
No lines between black and white.
No lines between young and old.
No lines between our side and your side.
No lines between me and you.

Not to draw lines is not to discriminate.
The ability to discriminate is the key to perception.
Are we taking leave of our senses?

Maybe.

Maybe we're learning to draw lines with disappearing ink.



Paul Williams, Das Energi, New York, Elektra, 1973

NOTES

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Other people use other signs for the marked and unmarked states. One useful one for the mark, when you are using a typewriter, is parentheses. () indicates the mark (() () (())) is an expression.

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Some folks are disturbed by this and call it and "empty space" or a "blank." (Kahout and Pinkava, 1980)

p. 2-3 For a full discussion, the reader is referred to P.L. Heath's article on "Nothing" in the Encyclopedia of Philosophy wherein we read (among other things)" "Ever since Parmenides laid it down that it is impossible to sp ak of what is not, broke his own rule in the act of stating it, and deduced him elf into a world where all that ever happened was nothing, the impression has persisted that the narrow path between sense and nonsense on this subject is a difficult one to tread and that altogeth r the less said of it the better."


There is an old Zen koan *
What is the sound of one hand clapping?
What is the form of one mark marking?
* A koan is a puzzle that is supposed to help you realize who you are.

And the contemporary philosopher Paul Williams.

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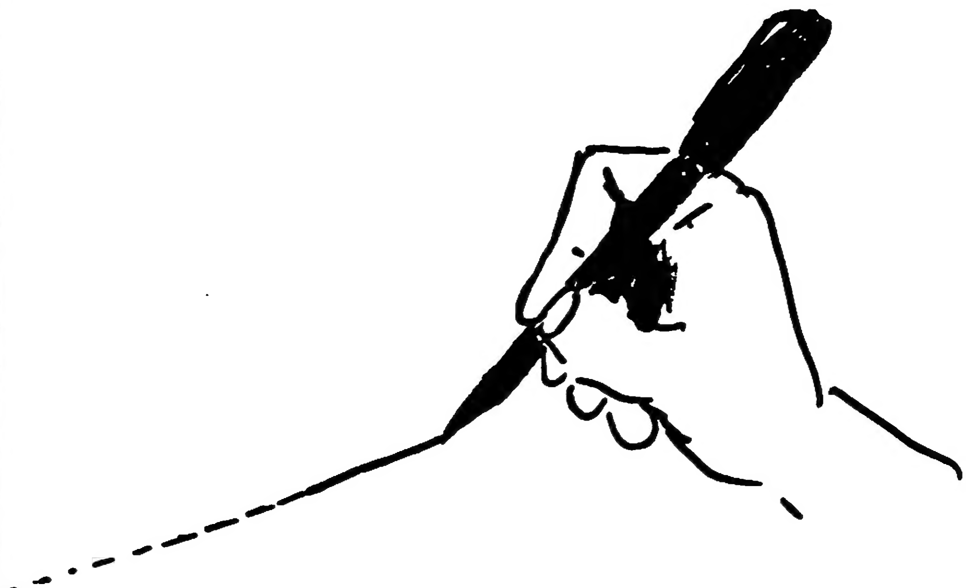

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